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Newton's Law of Motion

First Law –

An object at rest remains at rest, or if in motion remains in motion, unless acted upon by an external force.

Inertia: Tendency of a body to maintain its states of motion or rest

$$\boxed{\text{Inertia} \propto \text{Mass}}$$

$\uparrow$ Inertia $\Rightarrow$ Difficult to change state of body.

Types of Inertia –

1) Inertia of Rest
2) Inertia of Motion

Second Law —

net force

$$\boxed{\vec{F} = \frac{d\vec{p}}{dt}} \quad \text{where} \quad \boxed{\vec{p} = m\vec{v}}$$

$$\Rightarrow \vec{F} = \left(\frac{d\vec{p}}{dt}\right) = m\left(\frac{d\vec{v}}{dt}\right) + v\left(\frac{dm}{dt}\right)$$

★ If Mass Const. $\Rightarrow$ $\boxed{f = ma}$

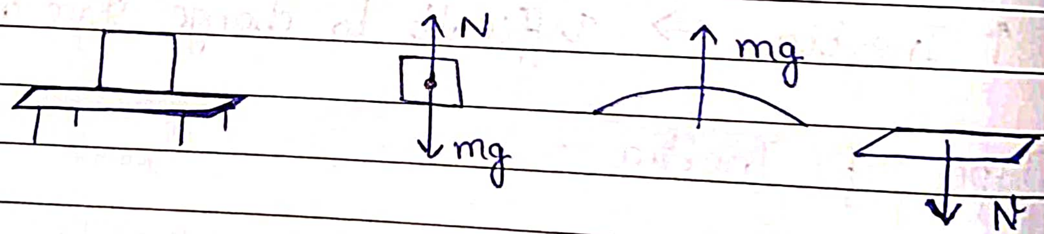
If Vel. Const. $\Rightarrow$ $\boxed{f = v\left(\frac{dm}{dt}\right)}$

Third Law —

To every action, there is an equal an opposite rx^n.

Action – Rx^n forces act on diff. bodies.

Eg:

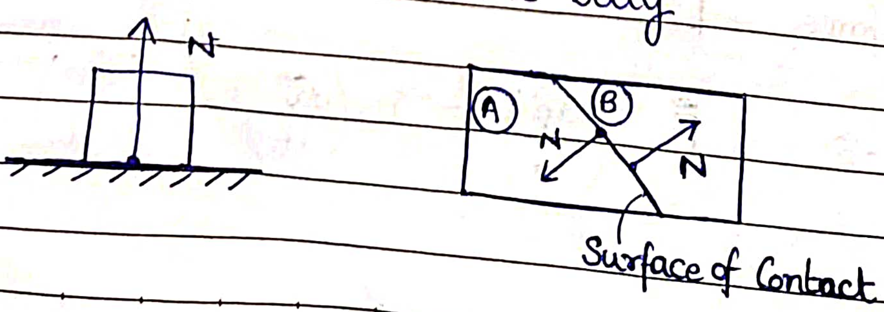


Block Earth Platform

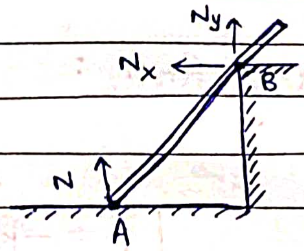
Types of Forces —

1) Normal : i) $\perp$ to Surface of Contact
ii) Towards the body

Eg:

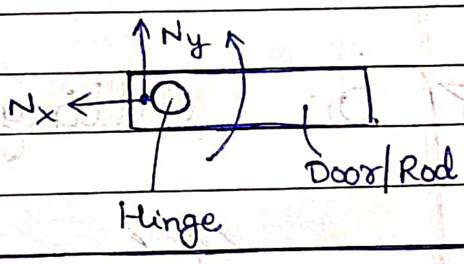


☆ In cases where pt/surface of contact not recognizable, we take N_x and N_y in x and y dirx^{n}s. Dirx^n of Normal force NOT clear.

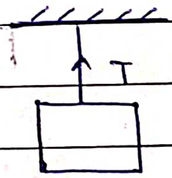


Eg: Ladder AB is kept on wall.
pt. B is where wall endpt.
We don't know dirx^n of Normal force at B.

Eg: Normal force by hinge on door/rod

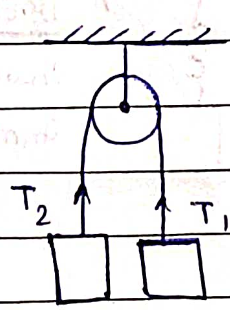


2) Tension: i) force due to string
ii) Away from body



☆ Note – Tension on massless string is same at all pts. on string (or massive string with $a = 0$)

☆ Note –



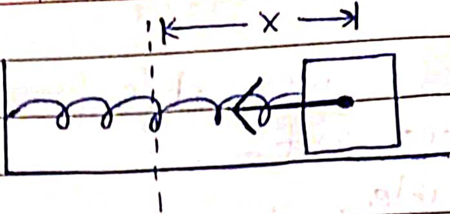
$\boxed{T_1 = T_2}$

iff

String is Massless

Pulley is Frictionless or Massless

3) Spring force :



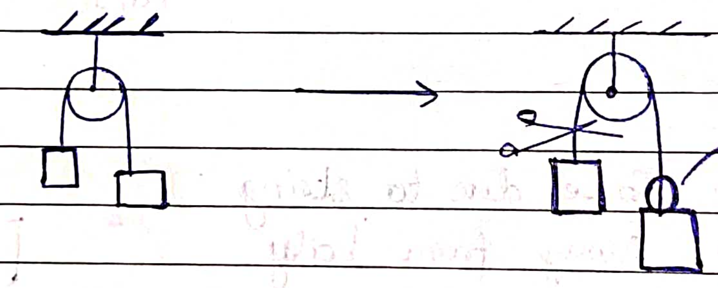
$$\boxed{\vec{F} = -k\vec{x}}$$

k = Spring Constant (N/m)

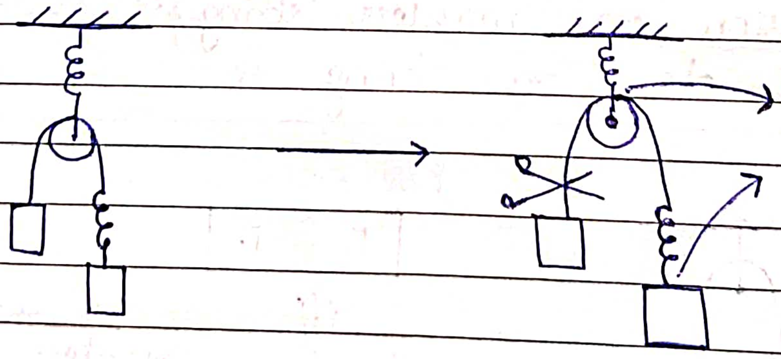
x = Extension or Compression in Spring

Value of K depends on Material & Natural Length of Spring

☆ On Cutting,

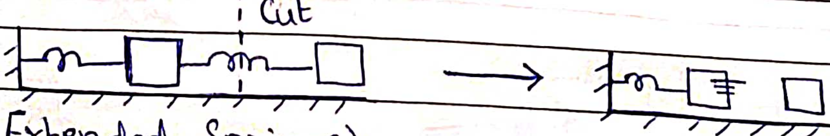


Tension changes immediately



Spring force DOESN'T change immediately

☆ If spring is cut, force due to it vanishes.

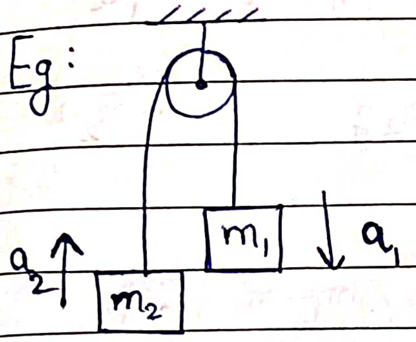


(Extended Springs)

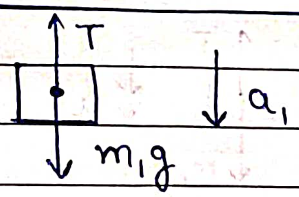
Free Body Diagram —

1) Write all force on body
2) Show dirxⁿ of acc.

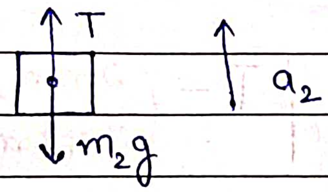
Eg:



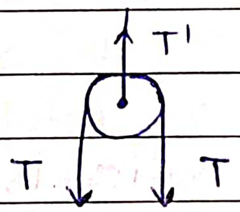
FBD of m_1



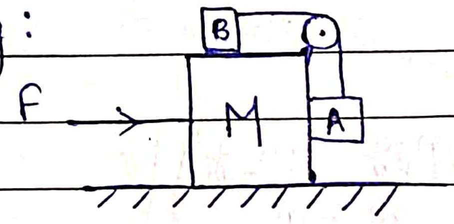
FBD of m_2



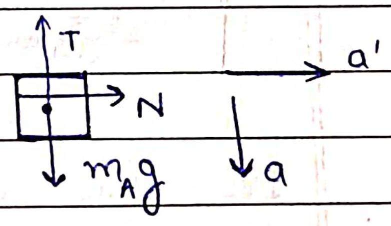
Pulley



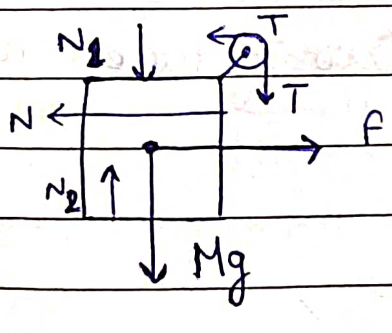
Eg:



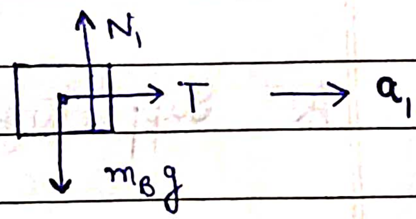
FBD of A



FBD of M



FBD of B



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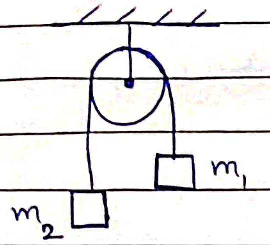
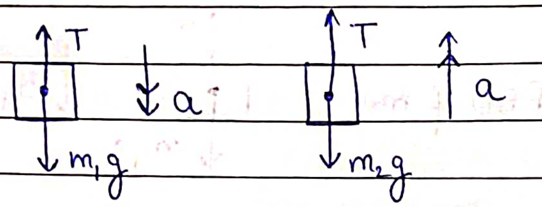
Q) $(m_1 > m_2)$ Find tension in string and acc. of masses



A)



$$m_1 g - T = m_1 a$$

$$m_2 g - T = -m_2 a$$

$$\boxed{T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g} \quad ; \quad \boxed{a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g}$$

☆ Short Trick –

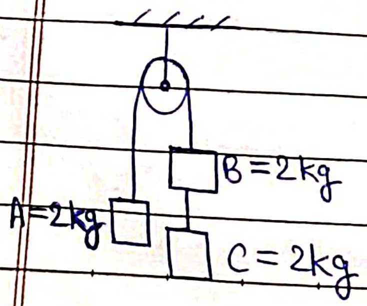
When mag. of acc. of **all** bodies is same,

$$\boxed{a = \left(\frac{F_{\text{Net system}}}{\sum \text{Mass of obj's.}}\right)} \leftarrow \left(\begin{array}{l} \text{Force } + + \text{ if supporting motion} \\ \text{Force } - - \text{ if opposing motion} \end{array}\right)$$

In this question, $\boxed{a = \left(\frac{m_1 g - m_2 g}{m_1 + m_2}\right)}$

As Supporting force $= m_1 g$ & Opposing force $= m_2 g$

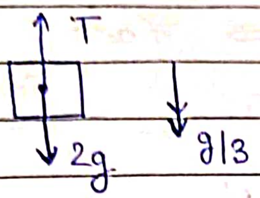
Q)



Find Tension in string connecting B & C

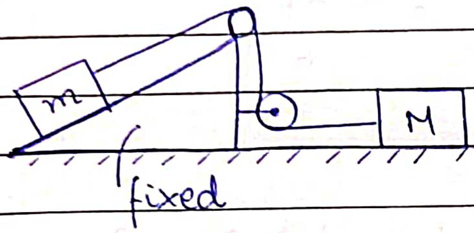
A) $a = \left(\dfrac{2g+2g-2g}{2+2+2}\right) \Rightarrow a = \left(\dfrac{g}{3}\right)$

F.B.D. of C



$\Rightarrow \dfrac{2g}{3} = 2g - T \Rightarrow \boxed{T = \dfrac{4g}{3}}$

Q)

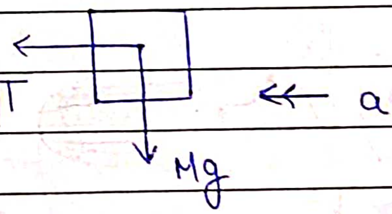


Find Tension in string.

A) Only force supporting motion is $mg s_\theta$, wt. of 'm' along incline.

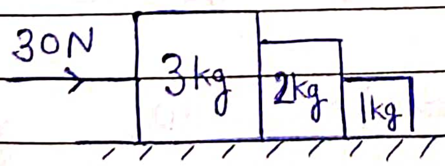
$a = \left(\dfrac{mg s_\theta}{M+m}\right)$

F.B.D. of M



$Ma = T \Rightarrow \boxed{T = \left(\dfrac{Mmg s_\theta}{M+m}\right)}$

Q)

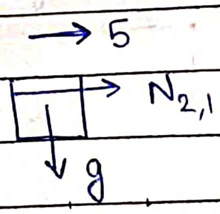


Find force applied by block 2 on block 1.

A) Only supporting force $= 30\ N$

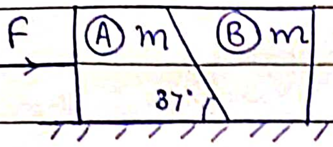
$a = \left(\dfrac{30}{3+2+1}\right) \Rightarrow a = 5$

FBD of 1



$N_{2,1} = 5 \cdot 1 \Rightarrow \boxed{N_{2,1} = 5}$

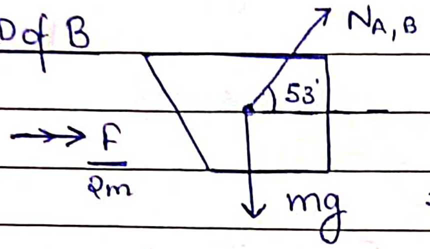
Date:

Q) 

Find force by A on B.

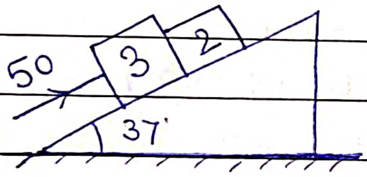
A) Supporting force : F $\Rightarrow$ $a = \dfrac{F}{2m}$

FBD of B



$N_{A,B}\, c_{53°} = m\left(\dfrac{F}{2m}\right)$

$\Rightarrow \boxed{N_{A,B} = \dfrac{5F}{6}}$

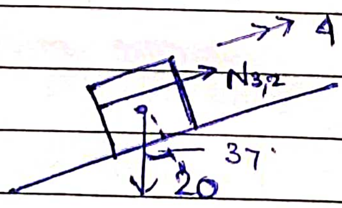
Q) 

Find force by 3 on 2.

A) ~~Supporting~~ forces : 50 N, $\underbrace{30\, s_{37°},\ 20\, s_{37°}}_{\text{Opposing}}$ (Wt. along incline)

$a = \left(\dfrac{50 - 30\, s_{37°} - 20\, s_{37°}}{5}\right) \Rightarrow a = 4$

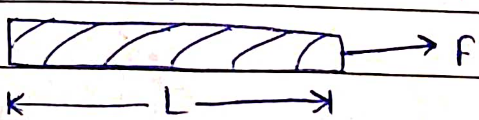
FBD of 2



$8 = N_{3,2} - 12$

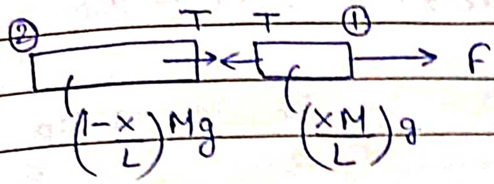
$\Rightarrow \boxed{N_{3,2} = 20}$

Q) Heavy Rope



Find tension at a dist. x from end where force is applied.

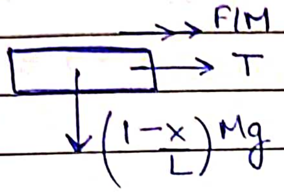
A) Consider as 2 seperate masses.



Supporting force : F

$a = F/M$

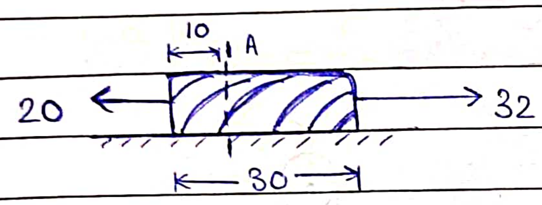
FBD of ②



$$T = \left(\frac{F}{M}\right)\left(1-\frac{x}{L}\right)M$$

$$\Rightarrow \boxed{T = F\left(1-\frac{x}{L}\right)}$$

Q)

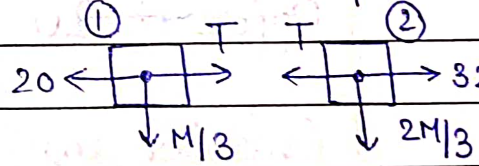


Find tension at line A

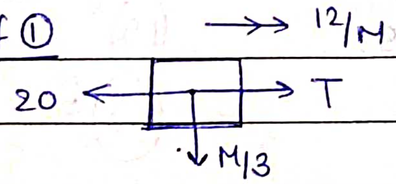
A) Supporting : 32 Opposing : 20

$$a = \left(\frac{32-20}{M}\right) \Rightarrow a = \frac{12}{M}$$

Consider as seperate obj.

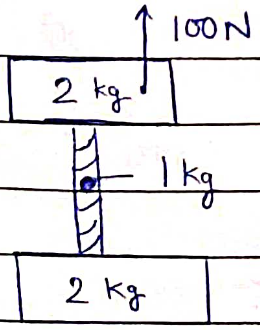


FBD of ①



$$T - 20 = \left(\frac{M}{3}\right)\left(\frac{12}{M}\right) \Rightarrow \boxed{T = 24}$$

Q)



If system going vertically up, find Tension in middle of rope.

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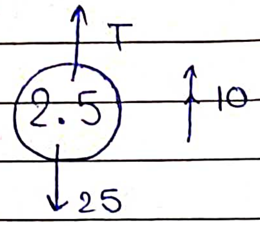
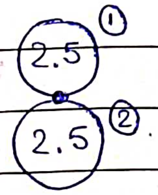
A) Support : 100 N   Oppose : 20 + 20 + 10 = 50 N (Wt. of all)

$a = \left(\frac{100-50}{2+2+1}\right) \Rightarrow a = 10$

Consider as seperate obj.s

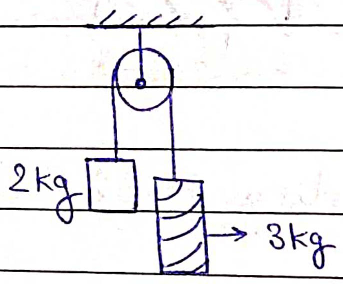
FBD of ②





$T - 25 = 25 \Rightarrow \boxed{T = 50}$

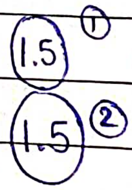
Q) Find tension at middle of rope.



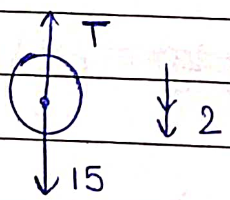
A) Support : 30
Oppose : 20

$\Rightarrow a = \left(\frac{30-20}{5}\right) \Rightarrow a = 2$

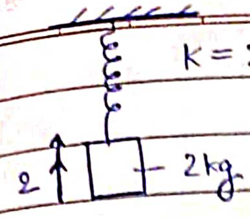
Consider as seperate objs.



FBD of ②

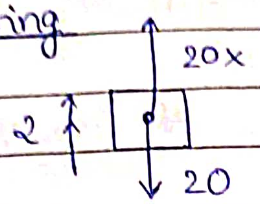


$15 - T = 2 \cdot (1.5) \Rightarrow \boxed{T = 12}$

Q) 

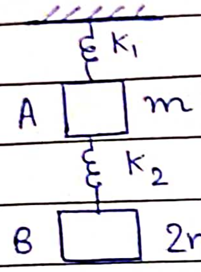
Find extension of spring.

A) FBD of Spring



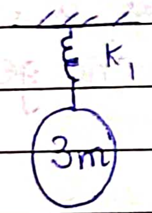
$20x - 20 = 2 \cdot 2$

$\Rightarrow \boxed{x = 1.2\ m}$

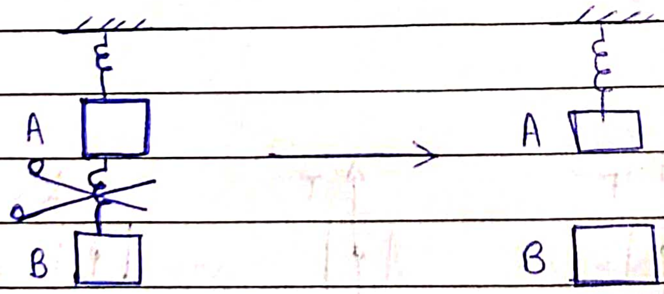
Q) 

Spring b/w A & B is cut.

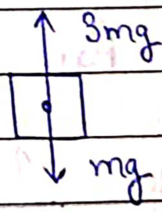
Find acc. of A & B just after cut.

A) 

$\Rightarrow \quad K_1 x_1 = 3mg$

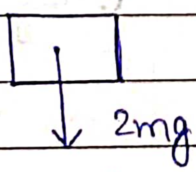


FBD of A



$\Rightarrow a_A = 2g$

FBD of B



$\boxed{a_B = g}$, $\boxed{a_A = 2g}$

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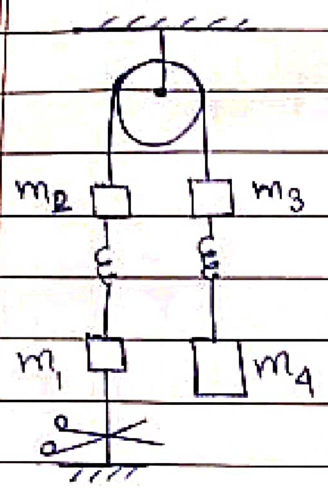
Date:

☆ Q)



$m_3 + m_4 > m_1 + m_2$

The string with floor is cut.

Find acc. of masses.

A) After Cutting,

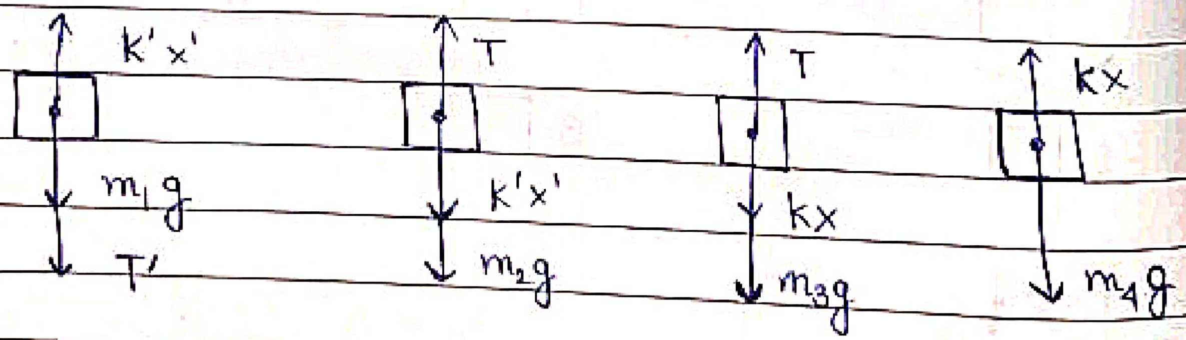
$a_4 = 0$; as Spring force in String m_3, m_4 doesn't change.

$a_3 = 0$; ☆ $\boxed{\text{Tension only changes when forces on String change}}$

$a_2 = 0$;

Since all forces same no acc.

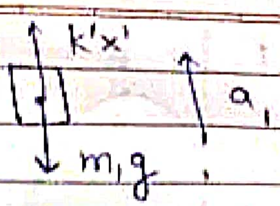
For m_1,



$kx = m_4 g \Rightarrow T = (m_4 + m_3) g$

$\Rightarrow k'x' = (m_4 + m_3 - m_2) g$

After cutting $T' = 0$,



$$a_1 = \frac{(m_4 + m_3 - m_2)g - m_1 g}{m_1}$$

$$\Rightarrow \boxed{a_1 = \frac{(m_4 + m_3 - m_2 - m_1)g}{m_1}}$$

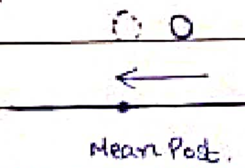
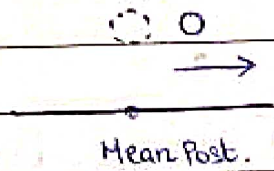
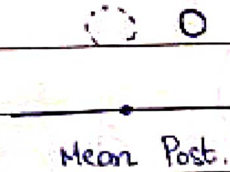
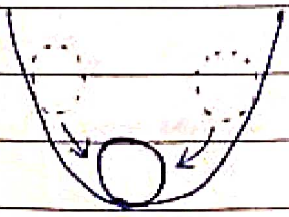
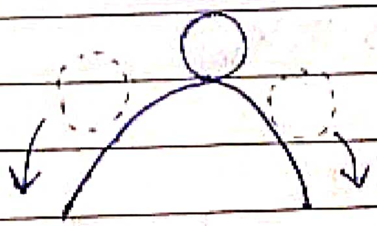
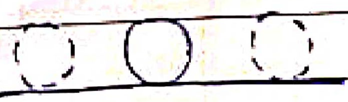
Equilibrium of Forces —

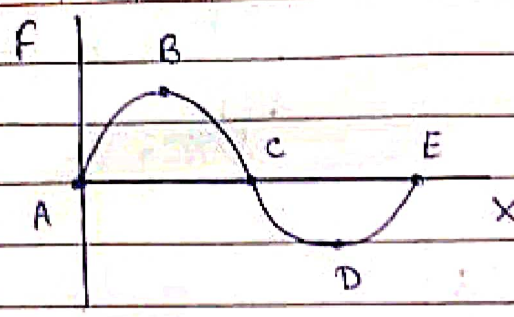
$\boxed{F_{net} = 0}$ ← Translational Equilibrium

$\boxed{\sum F_x = 0}$ $\boxed{\sum F_y = 0}$ $\boxed{\sum F_z = 0}$

(Mean Posn. is where $\sum F = 0$)

Types :

1) Stable	2) Unstable	3) Neutral
		
Body Returns to Mean posn. after disp.	Body away from Mean posn. after disp.	$F_{net} = 0$ at all pts in vicinity
		
F anti$\parallel$ x	$F \parallel x$	

Q) 

Find nature of pts.

A) For Equilibrium, $F_{net} = 0$

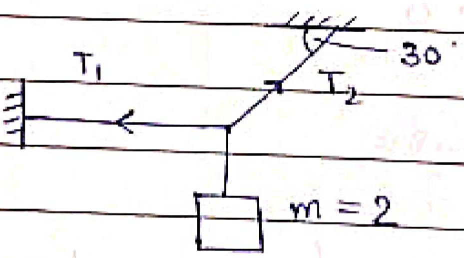
Equilibrium : A, C, E

NOT Equilibrium : B, D

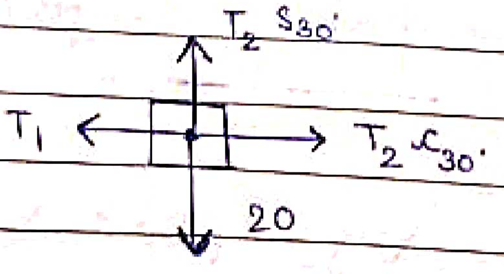
C – Stable $\left(\begin{array}{l} \Delta x > 0 \Rightarrow F < 0 \\ \Delta x < 0 \Rightarrow F > 0 \end{array} \right)$

$\Rightarrow$ Body returns to mean post. as F anti|| x

Similarly, A, E – Unstable

Q) 

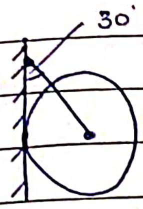
Find T_1 & T_2.

A) 

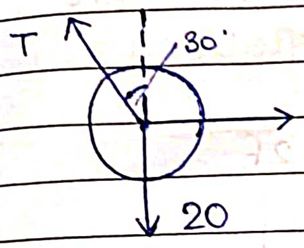
$T_2 s_{30°} = 20$

$T_1 = T_2 c_{30°}$

$\Rightarrow$ $\boxed{T_1 = 20\sqrt{3}}$, $\boxed{T_2 = 40}$

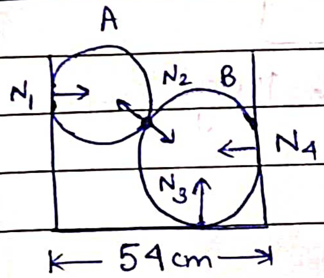
Q) 

$m = 2$ Find force by wall on ball.

A) 

$T c_{30°} = 20$

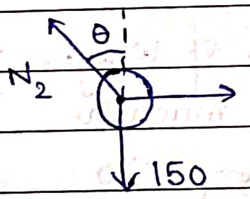
$T s_{30°} = N \Rightarrow \boxed{N = 20/\sqrt{3}}$

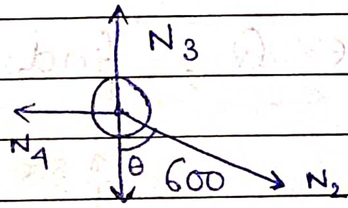
Q) 

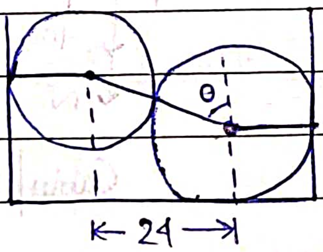
$r_A = 12$ cm $\quad r_B = 18$ cm

$m_A = 15$ kg $\quad m_B = 60$ kg

$N_1, N_2, N_3, N_4 = ?$

A) 



Now, 

We have $30 s_\theta = 24$

$\Rightarrow s_\theta = 4/5$

for A, $\quad 150 = N_2(3/5) \Rightarrow \boxed{N_2 = 250}$

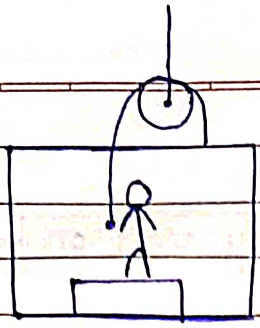
$N_1 = N_2(4/5) \Rightarrow \boxed{N_1 = 200}$

for B, $\quad N_4 = N_2(4/5) \Rightarrow \boxed{N_4 = 200}$

$N_3 = 600 + N_2(3/5) \Rightarrow \boxed{N_3 = 750}$

Date:

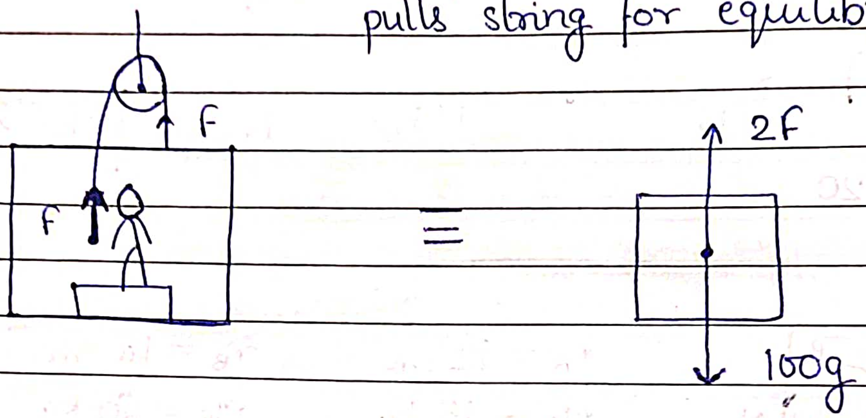
Q)



Mass of Man = 60
Mass of Cabin = 40

Find force with which man pulls string for equilibrium.

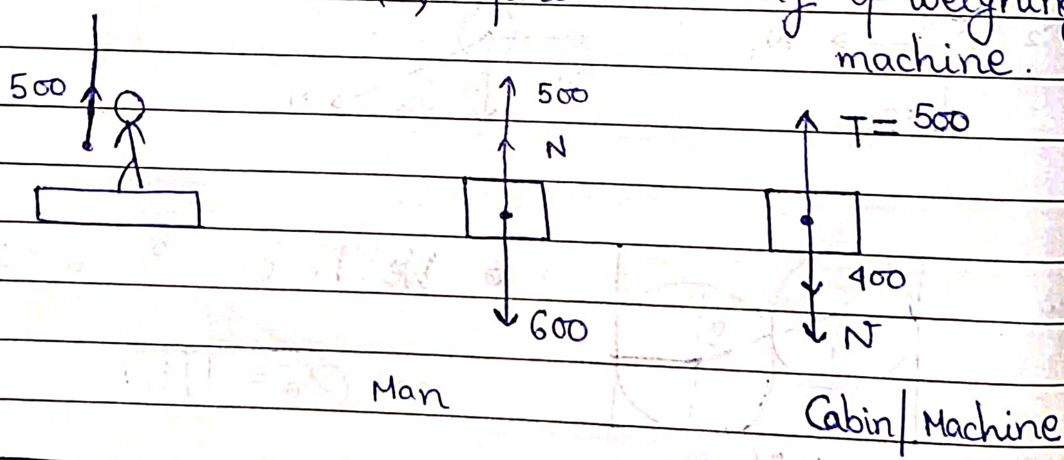
A)



$2F = 100g \Rightarrow \boxed{F = 500}$

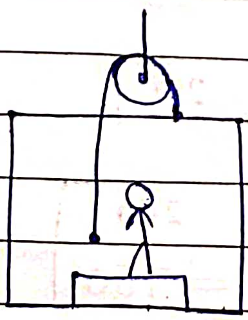
Q) In above Q, find reading of weighing machine.

A)



$\boxed{N = 100}$

Q)

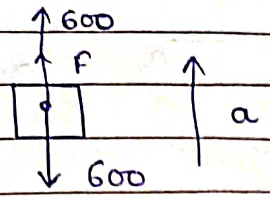


Mass of Man = 60
Mass of Cabin = 40

Find force if machine shows correct reading.

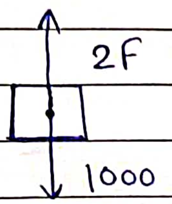
A)

Man



$f = 60a$

System (Man+Cabin)



$2f - 1000 = 100a$

Solving, $\boxed{a = 50}$, $\boxed{f = 3000}$

Inertial & Non-Inertial Frames –

☆ $\boxed{\text{Frame} = \text{Coordinate system with observer at origin}}$

Inertial frame : 1) Observer at rest or const. vel.
2) $f = ma$ is valid.

Non-inertial frame : 1) Observer moving with acc.
2) $f = ma$ is NOT valid

Pseudo force : fictitious force seen by observer in non-inertial frame.

☆ Pseudo force on obj.s depends ONLY on acc. of observer. It imparts an additional acc. to obj.s with mag. $\boxed{a = a_{observer}}$ and dirx^n opp. to observer's motion.

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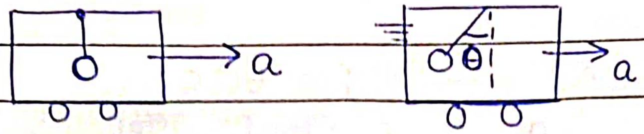
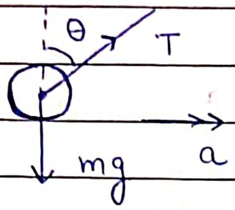
Q)  Find final angle θ.

A) From Ground's frame,

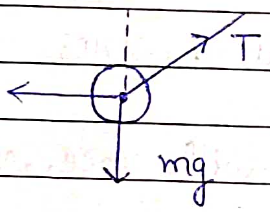
FBD 

$T c_\theta = mg$

$T s_\theta = ma$

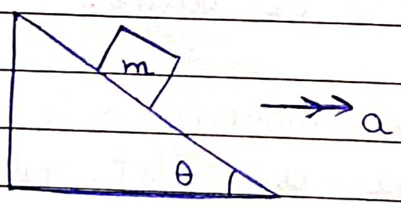
$\Rightarrow \boxed{\theta = \tan^{-1}(a/g)}$

From Car's frame,

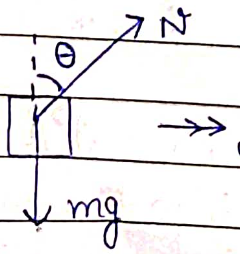
FBD 

ma (Pseudo)

(Same eq^{n}s)

Q)  The whole system acc with 'a'. If m at rest w.r.t incline, find 'a'

A) From Ground's frame,

FBD 

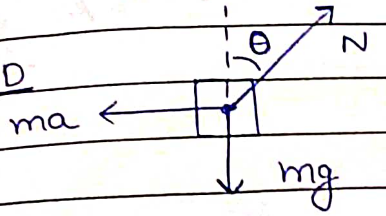
$N s_\theta = ma$

$N c_\theta = mg$

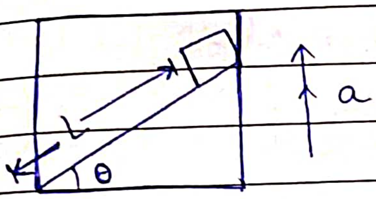
$\Rightarrow \boxed{a = g t_\theta}$

from Incline's frame,

FBD

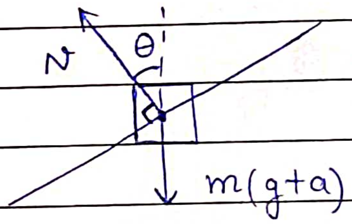


(Same eqⁿs)

Q) 

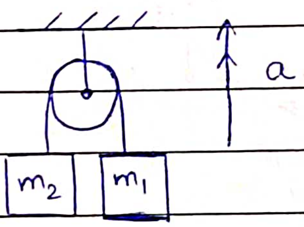
Find time taken to reach bottom.

A) From cabin's frame,



$a_{\text{along incline}} = (g+a)\,s_\theta$

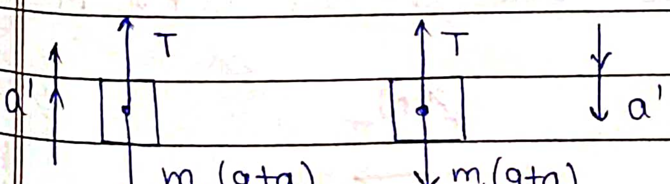
$\Rightarrow \boxed{t = \sqrt{\dfrac{2L}{(g+a)s_\theta}}}$

Q) 

System with acc. 'a'.

Find tension if $m_1 > m_2$.

A) In pulley's frame,



$a' = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)(g+a)$

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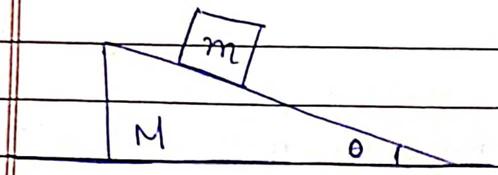
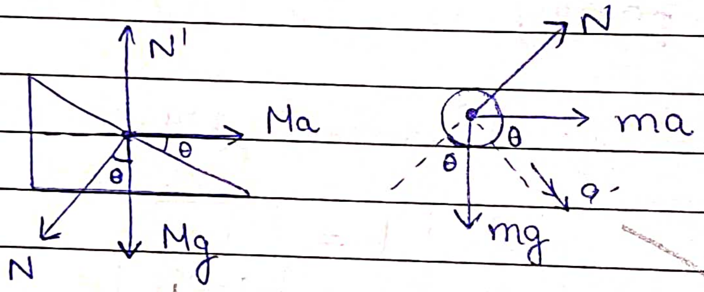
for 'm_1', $\quad m_1(g+a) - T = m_1\left(\frac{m_1-m_2}{m_1+m_2}\right)(g+a)$

$$\Rightarrow \boxed{T = \left(\frac{2m_1m_2}{m_1+m_2}\right)(g+a)}$$

Q) 

All surfaces smooth. Find acc. of M.

A) In M's frame,



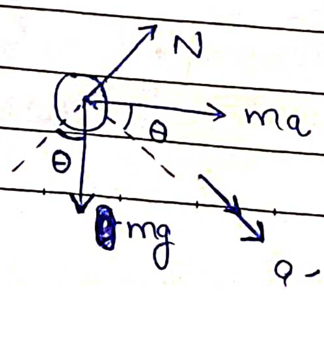
We have, $\quad N s_\theta = Ma \quad , \quad N = mg\, c_\theta - ma\, s_\theta$

$$\Rightarrow Ma = mg\, c_\theta s_\theta - ma\, s_\theta^2$$

$$\Rightarrow \boxed{a = \left(\frac{mg\, c_\theta s_\theta}{m s_\theta^2 + M}\right)}$$

Q) In above Q, find acc. of m w.r.t. M.

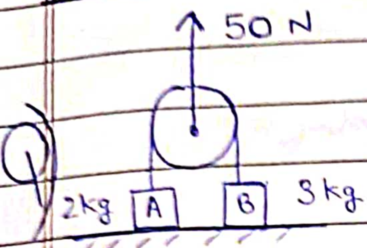
A) In M's frame,



$$ma c_\theta + mg s_\theta = ma'$$

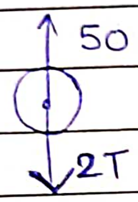
$$\Rightarrow \boxed{a' = g s_\theta + \left(\frac{mg c_\theta s_\theta}{m s_\theta^2 + M}\right)}$$

Q)



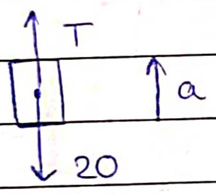
Find acc. of both blocks.

A) Pulley



$\Rightarrow \quad 2T = 50 \Rightarrow T = 25$

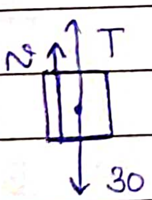
A



Since $T > 20$, block lifts

$2a = T - 20 \Rightarrow \boxed{a_A = 2.5}$

B

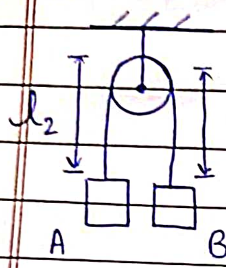


Since $T < 30$, block NOT lift.

$\Rightarrow \boxed{a_B = 0}$

Constraint Relⁿ —

1) String Constraint : Length of String Const.

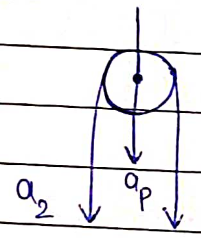


$l_1 + l_2 = \text{Const.}$

$\Rightarrow v_A + v_A = 0$

$\Rightarrow \boxed{a_A = -a_B}$

☆



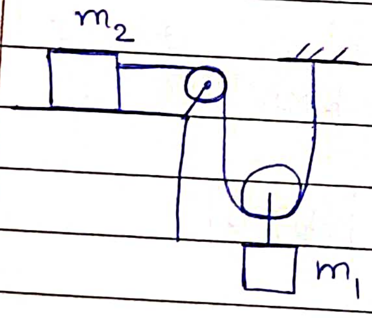
(Works iff a_1 & $a_2 \parallel a_p$)

$$\boxed{a_p = \left(\frac{a_1 + a_2}{2}\right)}$$

Proof: In Pulley frame,

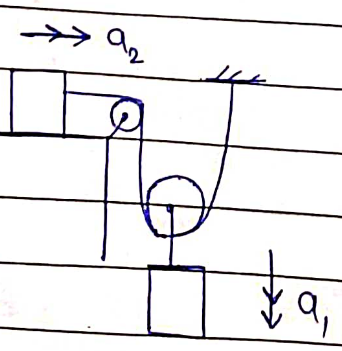
$(a_1 - a_p) = (a_p - a_2)$

Q)



Find acc. of m_1 & m_2.

A)

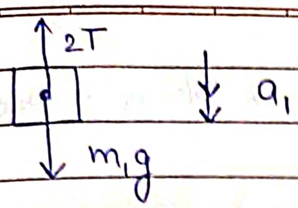


Now, $a_p = a_1$ &

$a_p = \left(\frac{a_2 + 0}{2}\right) \Rightarrow a_p = \left(\frac{a_2}{2}\right)$

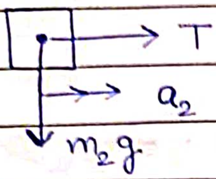
$\Rightarrow a_2 = 2a_1$

m_1



$$m_1 g - 2T = m_1 a_1$$

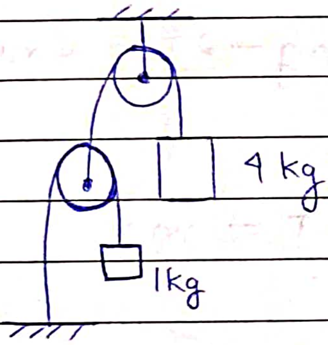
m_2



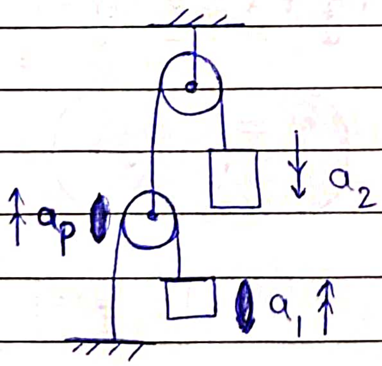
$$T = m_2 a_2$$

Now, $m_1 g - 2m_2 a_2 = m_1 a_1$

$$\Rightarrow \boxed{a_1 = \left(\frac{m_1 g}{4m_2 + m_1}\right)}, \quad \boxed{a_2 = \left(\frac{2m_1 g}{4m_2 + m_1}\right)}$$

Q) 

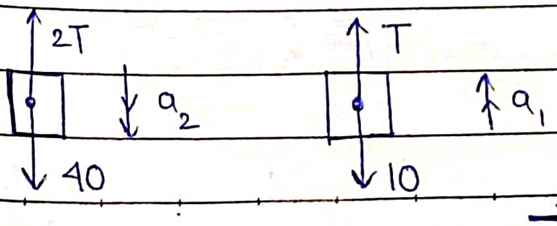
Find acc. of both masses.

A) 

$$a_2 = a_p$$

$$a_p = \left(\frac{0 + a_1}{2}\right) \Rightarrow a_p = \frac{a_1}{2}$$

$$\Rightarrow a_1 = 2a_2$$



$$40 - 2T = 4a_2$$

$$T - 10 = a_1$$

$$\Rightarrow \boxed{a_2 = 2.5}, \quad \boxed{a_1 = 5}$$

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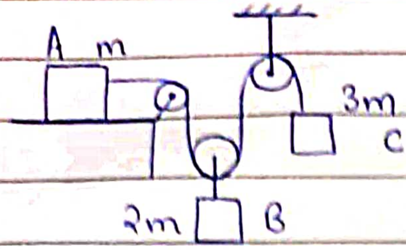
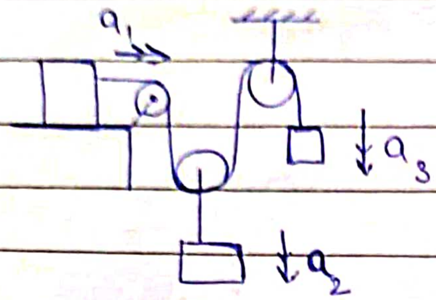
Q) 

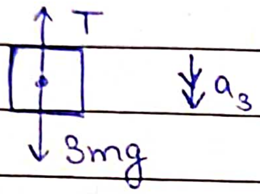
Find acc. of all blocks

A) 

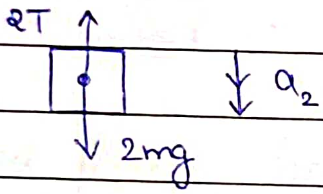
We have,

$$a_2 = \left(\frac{a_1 - a_3}{2}\right)$$

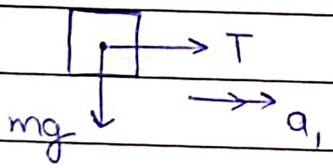
$$\Rightarrow a_1 - 2a_2 - a_3 = 0$$

For C, 

$$3mg - T = 3ma_3$$

for B, 

$$2mg - 2T = 2ma_2$$

for A, 

$$T = ma_1$$

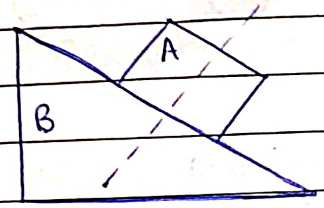
$$m(a_1 - 2a_2 - a_3) = 0 \Rightarrow T - 2mg + 2T - mg + \frac{T}{3} = 0$$

$$\Rightarrow \frac{10T}{3} = 3mg \Rightarrow \boxed{T = \left(\frac{9mg}{10}\right)}$$

Solving We get, $\boxed{a_1 = 9}$, $\boxed{a_2 = 1}$, $\boxed{a_3 = 7}$

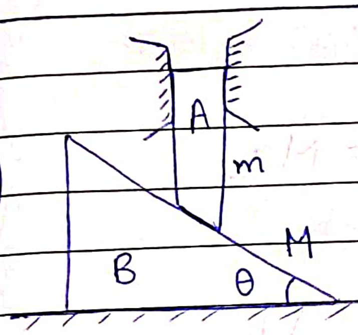
2) Wedge Constraint :

When 2 bodies in contact always, their rel. vel./acc. = 0 along normal, otherwise they seperate.



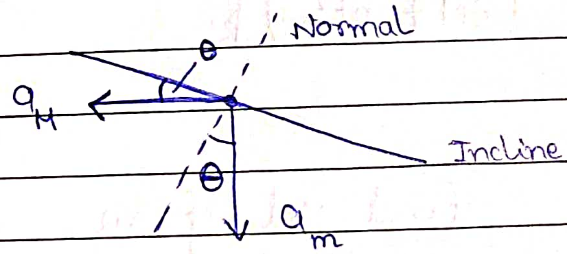
$$\boxed{\begin{aligned}(V_A)_{normal} &= (V_B)_{Normal}\\ (a_A)_{Normal} &= (a_B)_{Normal}\end{aligned}}$$

Q)



Find acc. of A & B if A can only move downwards.

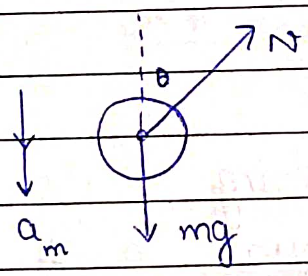
A)



$$a_M s_\theta = a_m c_\theta$$

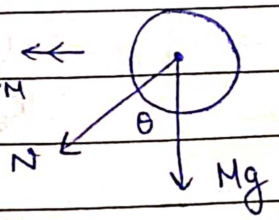
$$\Rightarrow a_m = a_M t_\theta$$

for A,



$$mg - N c_\theta = m a_m$$

for B, a_M

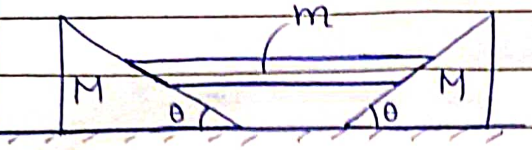


$$N s_\theta = M a_M$$

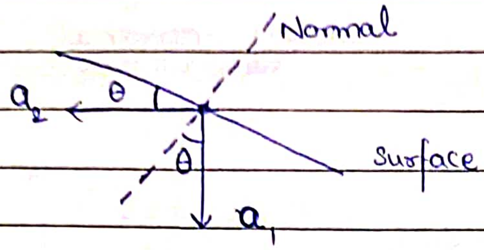
On solving,

$$\boxed{a_m = \left(\frac{mg t_\theta^2}{M + m t_\theta^2}\right)}$$

$$\boxed{a_M = \left(\frac{mg t_\theta}{M + m t_\theta^2}\right)}$$

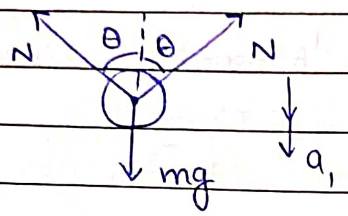
Q) 

Find acc. of m.

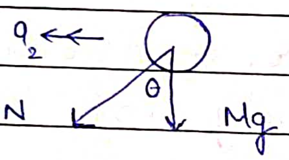
A) 

$a_2 s_\theta = a_1 c_\theta$

$\Rightarrow \boxed{a_2 = a_1 T_\theta}$

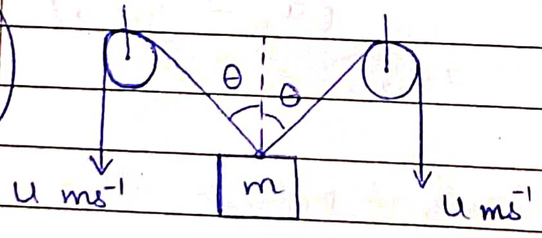
for m, 

$mg - 2N c_\theta = m a_1$

for M, 

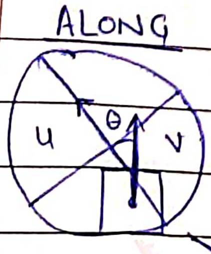
$N s_\theta = M a_2$

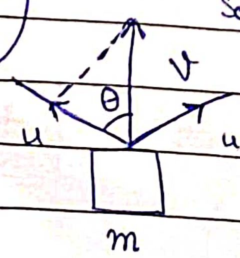
On solving, $\boxed{a_1 = \left(\dfrac{mg\, t^2_\theta}{2M + m t^2_\theta}\right)}$

☆ Q) 

Find vel. of m.

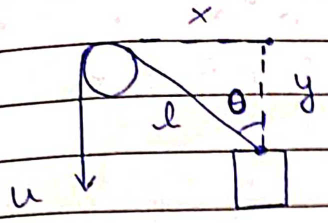
A) Since m connected to string, its vel. ALONG string should equal to 'u'.





So $v c_\theta = u \Rightarrow \boxed{v = \left(\dfrac{u}{c_\theta}\right)}$

Another Method -



$$x^2 + y^2 = l^2$$

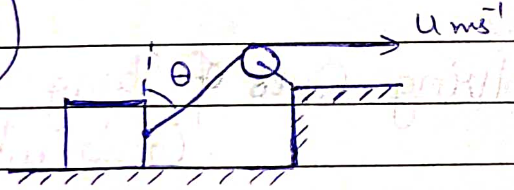
$$\cancel{\Rightarrow 0 + 2y\,y' =}$$

$$\Rightarrow 0 + 2y\dot{y} = 2l\dot{l}$$

$$\Rightarrow \dot{y} = \left(\frac{l}{y}\right)\dot{l} \Rightarrow \boxed{v = \frac{u}{c_\theta}}$$

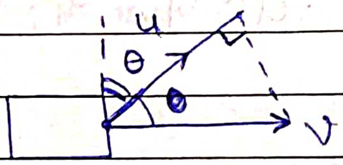
☆ **ALWAYS** take component of REAL motion. Vel. of objs joined by string = along string.

Q)



Find vel. of block.

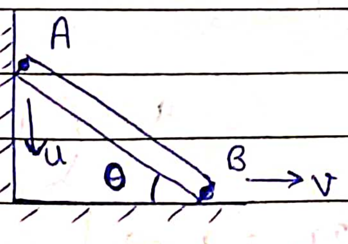
A)



$$v s_\theta = u$$

$$\Rightarrow \boxed{v = u c_\theta}$$

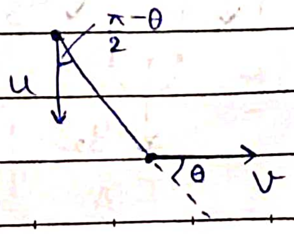
Q)



Find relⁿ b/w v & u

A) Length of Rod Const.

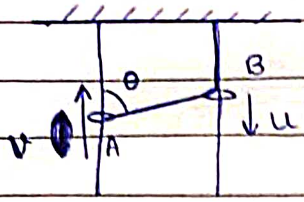
$\Rightarrow$ $v_{rel.}$ along rod $= 0$



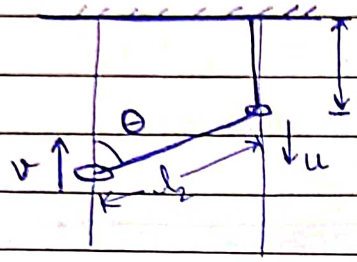
$$u s_\theta = v c_\theta$$

$$\Rightarrow \boxed{v = u t_\theta}$$

Date:

★Q) 

Find rel^n b/w v & u
String thru A to B to top.

A) 

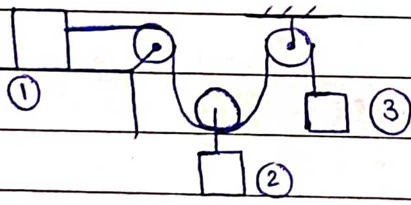
We have, $\dot{l}_1 = u$
and $\dot{l}_2 = -(v+u)c_\theta$

Also, $l_1 + l_2 = \text{Const.}$

$$\Rightarrow \dot{l}_1 + \dot{l}_2 = 0 \Rightarrow u - (v+u)\,c_\theta = 0$$

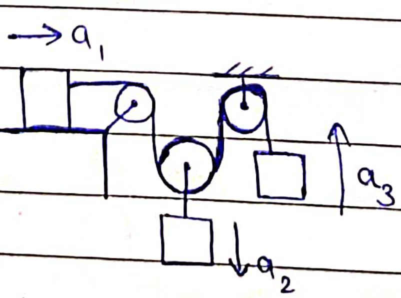
$$\Rightarrow \boxed{u(1 - c_\theta) = (+v \cdot c_\theta)}$$

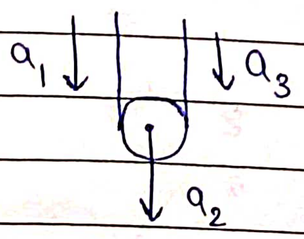
★ Various Ways of Solving Ques of String Constraint

Q) 

Find rel^n b/w a_1, a_2, a_3.

A) M-1 : Pulley

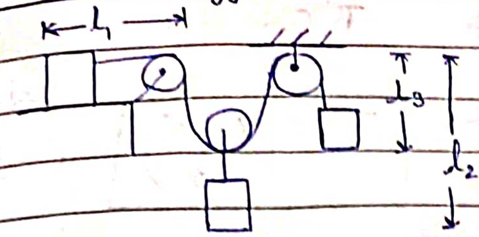


$\equiv$ 

$$a_2 = \left(\frac{a_1 + a_3}{2}\right)$$

$$\Rightarrow \boxed{2a_2 = a_1 + a_3}$$

M-2: Diff.



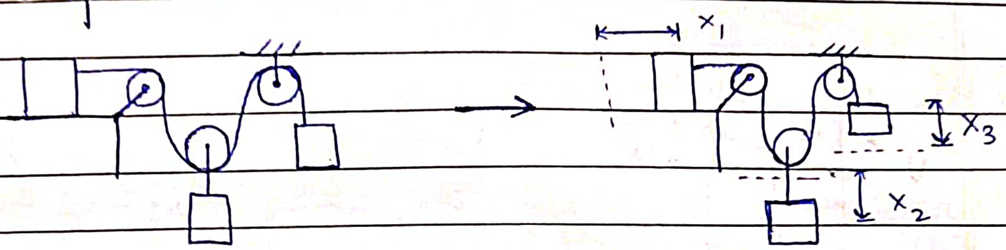
$l_1 + 2l_2 + l_3 = 0$

$\Rightarrow \ddot{l}_1 + 2\ddot{l}_2 + \ddot{l}_3 = 0$

$\Rightarrow (-a_1) + 2a_2 + (-a_3) = 0$

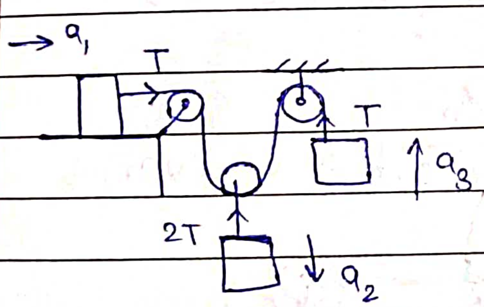
$\Rightarrow \boxed{2a_2 = a_1 + a_3}$

M-3: Disp.



Now, $2x_2 = x_1 + x_3 \Rightarrow \boxed{2a_2 = a_1 + a_3}$

M-4: $\sum T \cdot a = 0$ (Virtual Work done by Internal force)

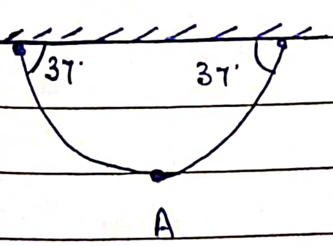


$Ta_1 - 2Ta_2 + Ta_3 = 0$

$\Rightarrow \boxed{2a_2 = a_1 + a_3}$

21/6/22

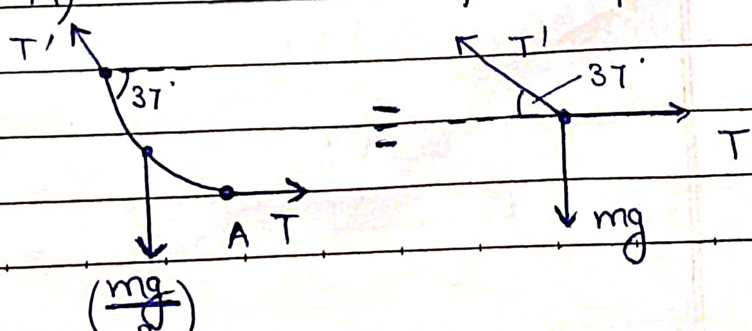
Q)



Find tension at A

Mass of rope = m.

A) We make FBD of 1/2 rope.



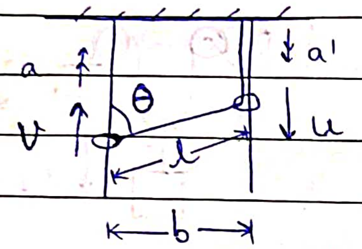
Date:

So, $T' s_{37} = \left(\frac{mg}{2}\right)$ & $T' c_{37} = T$

$$\Rightarrow T = \left(\frac{mg}{2}\right) T_{37} \Rightarrow \boxed{T = \left(\frac{2mg}{3}\right)}$$

24/6/22

Q)



Find relⁿ b/w a & a'

A) Earlier we found $v = \left(\frac{1 - c_\theta}{c_\theta}\right) u$

$$\Rightarrow \boxed{\left(\frac{dv}{dt}\right) = \left(\frac{du}{dt}\right)(S_\theta - 1) + u\,(S_\theta t_\theta)\left(\frac{d\theta}{dt}\right)} \quad \text{— (i)}$$

We know, $l = b / s_\theta$

$$\Rightarrow \dot{l} = (-C_\theta T_\theta)\, b\, \dot{\theta} = -(v+u) c_\theta$$

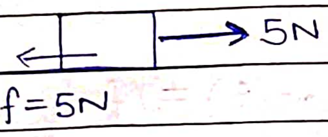
$$\Rightarrow \boxed{\dot{\theta} = \frac{+(v+u) s_\theta^2}{b}} \quad \text{— (ii)}$$

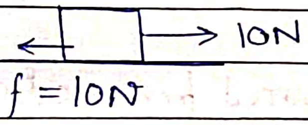
Now (ii) → (i) ⇒ $\boxed{a = a'(S_\theta - 1) + \left(\frac{u(v+u) s_\theta^3}{b c_\theta^2}\right)}$

Friction –

Static friction: Friction on body at rest.
It is self adjusting

Eg:



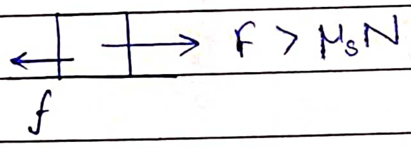


☆ Max. value of static friction = Limiting friction

$$\boxed{f_{s_{max.}} = \mu_s \underbrace{N}_{\text{Normal}}} \quad ; \quad \mu_s = \text{Coeff. of Static friction}$$

$$\Rightarrow \quad f_s \leq \mu_s N$$

Kinetic friction: Friction on body in motion.
It is fixed.

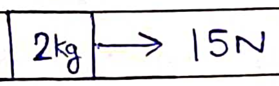


$$\boxed{f = \mu_k N}$$

Coeff. of Kinetic friction.

☆ $\boxed{\mu_k < \mu_s}$

Q)



$\mu_s = 0.5$, $\mu_k = 0.4$

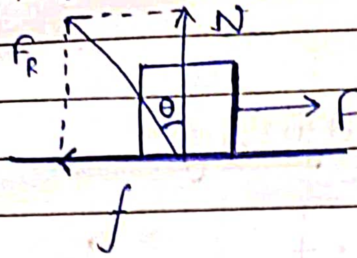
Find friction on body.

A) $f = 15 > 10 = \mu_s N$

$\Rightarrow f = \mu_k N = (0.4)(20)$

$\Rightarrow \boxed{f = 8N}$

$N, f \rightarrow$ Contact forces.



✓ f_R = Net Contact force

✓ $f_R = \sqrt{N^2 + f^2}$

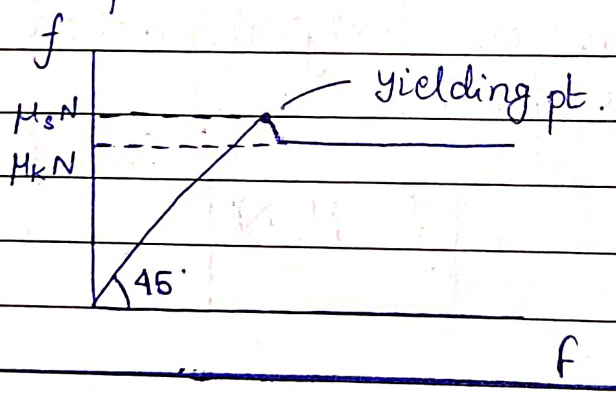
(∠ measured from vertical) ✓ $\tan(\theta) = \left(\frac{f}{N}\right)$

Since f has a max. limit $\Rightarrow$ θ has max. limit.

$\theta_{max} = \theta_s$ (angle of friction) $\Rightarrow$ $\mu_s = \tan(\theta_s)$

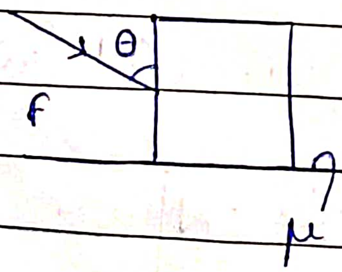
$\tan(\theta_s) = f_{max}/N$

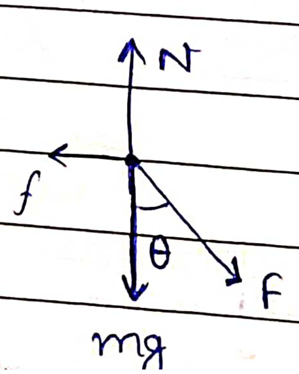
Graph –



Pushing & Pulling :

✓ Push :





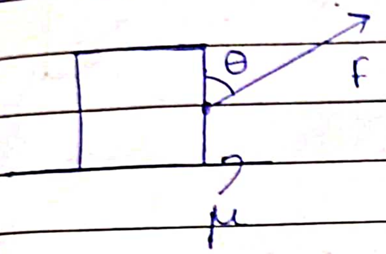
$N = F c_\theta + mg$

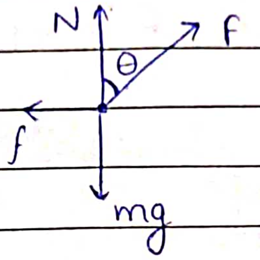
Obj moves if, $\quad F s_\theta \geq \mu_s (F c_\theta + mg)$

$$\Rightarrow \boxed{F \geq \left(\frac{\mu_s mg}{s_\theta - \mu_s c_\theta}\right)}$$

Now, $(s_\theta - \mu_s c_\theta) > 0 \Rightarrow t_\theta > \mu_s \Rightarrow \theta > \theta_s$

Pull :





$N = mg - F c_\theta$

Obj moves if, $\quad F s_\theta \geq \mu_s (mg - F c_\theta)$

$$\Rightarrow \boxed{F \geq \left(\frac{\mu_s mg}{s_\theta + \mu_s c_\theta}\right)}$$

$$\Rightarrow F_{min}(\theta) = \left(\frac{\mu_s mg}{s_\theta + \mu_s c_\theta}\right)$$

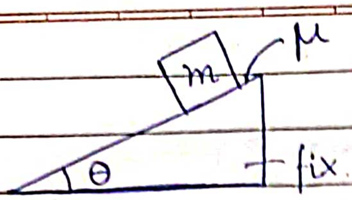
Now, $\quad s_\theta + \mu_s c_\theta \leq \sqrt{\mu_s^2 + 1} \quad (= \text{ at } \tan(\theta) = 1/\mu_s)$

$$\Rightarrow F_{min}(\theta) = \left(\frac{\mu_s mg}{s_\theta + \mu_s c_\theta}\right) \geq \left(\frac{\mu_s mg}{\sqrt{\mu_s^2 + 1}}\right)$$

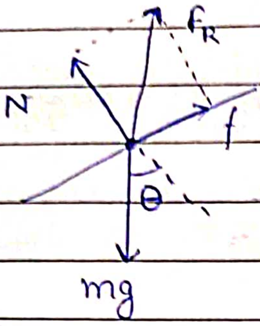
$\Rightarrow$ Min. of $\boxed{F_{min}(\theta) = \left(\frac{\mu_s mg}{\sqrt{\mu_s^2 + 1}}\right)}$ at $\boxed{\tan(\theta) = 1/\mu_s}$

☆ It is easier to pull than to push.

Date:

Q) 

If body not moving, find force by incline on body.

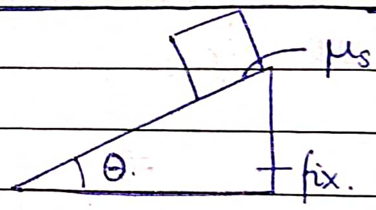
A) 

Since body at rest,

$f = mg s_\theta$, $N = mg c_\theta$

$F_R = \sqrt{f^2 + N^2} \Rightarrow \boxed{F_R = mg}$

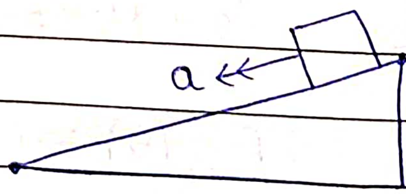
Q ☆ At $\theta = \theta_R$, body just starts to move



$\Rightarrow \underbrace{\boxed{\theta_R = \tan^{-1}(\mu_s)}}$ Angle of repose

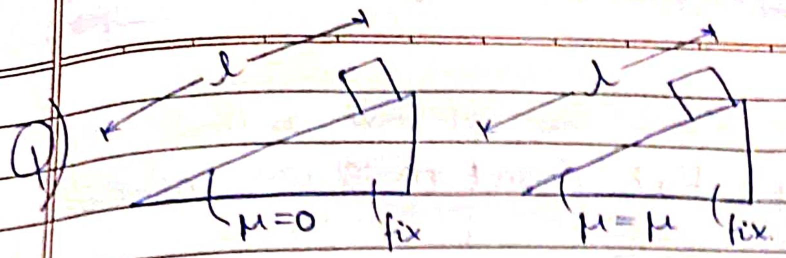
Q If $\theta > \theta_R$, body slides down with

$\boxed{a = g(s_\theta - \mu c_\theta)}$



If l is length of incline, time to slide down

$$\boxed{t = \sqrt{\frac{2l}{g(s_\theta - \mu c_\theta)}}}$$

1) 

If on incline 1, it takes time 't' and on incline 2, it takes time nt. Find μ.

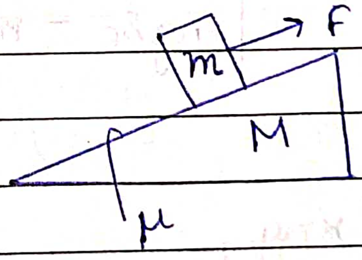
A) $t = \sqrt{\dfrac{2l}{g s_\theta}}$, $nt = \sqrt{\dfrac{2l}{g(s_\theta - \mu c_\theta)}}$

$\Rightarrow g s_\theta = \left(\dfrac{2l}{t^2}\right)$, $g s_\theta - (g c_\theta)\mu = \left(\dfrac{2l}{t^2}\right)\left(\dfrac{1}{n^2}\right)$

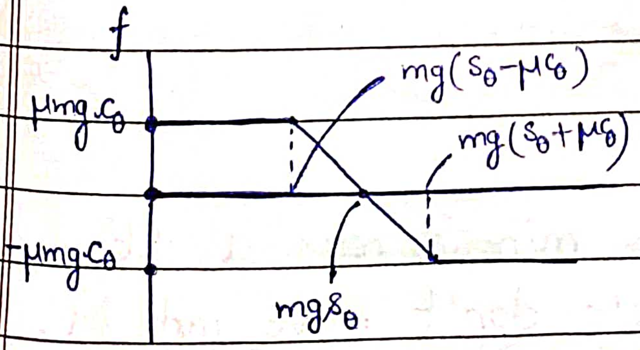
$\Rightarrow g s_\theta - (g c_\theta)\mu = \left(\dfrac{g s_\theta}{n^2}\right) \Rightarrow \boxed{\mu = \left(1 - \dfrac{1}{n^2}\right) t_\theta}$

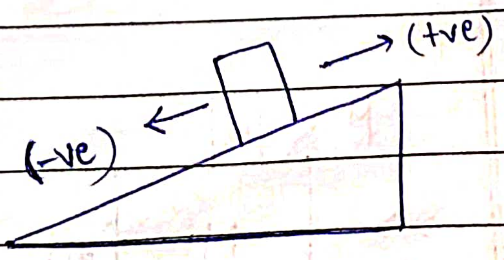
(28/6/22)

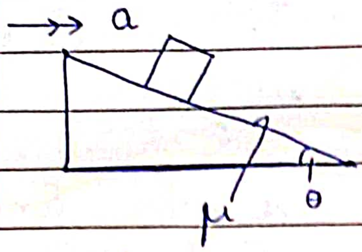
☆ If $F \geq mg(s_\theta - \mu c_\theta)$ & $F \leq mg(s_\theta + \mu c_\theta)$, the m does NOT move.



Proof : Block doesn't move if $|F - mg s_\theta| \leq f_{max} = \mu mg c_\theta$

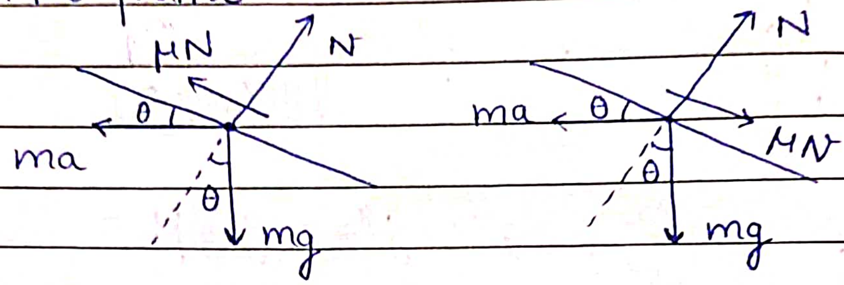




Q) 

Find min. & max. so that block doesn't move wrt. incline.

A) In M's frame

(Down)  (Up)

We want, $f_{\text{along incline}} = \text{friction}$

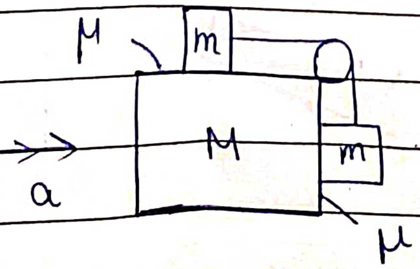
$$\Rightarrow \quad |mg s_\theta - ma c_\theta| = \mu N \quad (\text{for max. \& min } a)$$

We have, $N = mg c_\theta + ma s_\theta$

$$\Rightarrow \quad |mg s_\theta - ma c_\theta| = (mg c_\theta + ma s_\theta)(\mu)$$

On solving,

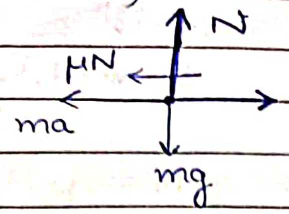
$$\boxed{a_{min.} = (g)\left(\frac{t_\theta - \mu}{1 + \mu t_\theta}\right)} \; ; \; \boxed{a_{max.} = (g)\left(\frac{t_\theta + \mu}{1 - \mu t_\theta}\right)}$$

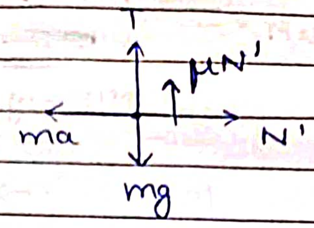
Q) 

Find min. & max. a s.t. block's don't move wrt. M.

In M's frame,

(Down)





$N = mg$ $\qquad\qquad$ $N' = ma$

$T = ma + \mu N$ $\qquad\qquad$ $T = mg - \mu N'$

$\Rightarrow T = ma + \mu mg$ $\qquad\qquad$ $\Rightarrow T = mg - \mu ma$

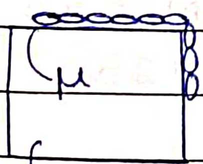
$$\Rightarrow \quad ma + \mu mg = mg - \mu ma$$

$$\Rightarrow \quad \boxed{a_{min} = (g)\left(\frac{1-\mu}{1+\mu}\right)}$$

☆ Since objs moving down, we get a_{min}. When $a\uparrow$, objs start moving up, we get a_{max}.

Similarly, $\boxed{a_{max.} = (g)\left(\frac{1+\mu}{1-\mu}\right)}$

Q)

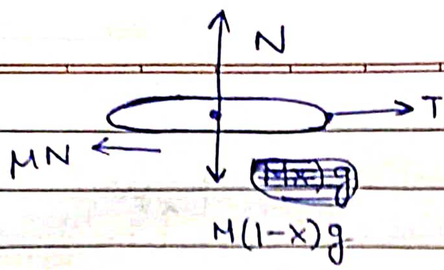


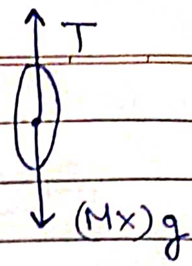
fix.

Find max. fraction of chain that can be hanged.

A) Let 'x' fraction be hanging.

Date:





$$N = M(1-x)g \Rightarrow \mu N = \mu M(1-x)g$$

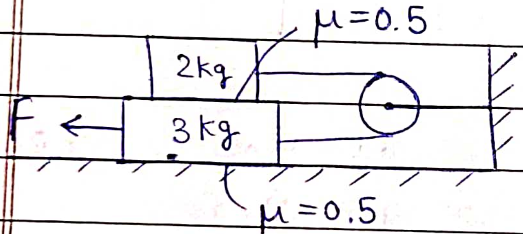
Now, $\mu N = T$, $T = (Mx)g$

$$\Rightarrow \mu Mg(1-x) = Mgx$$

$$\Rightarrow \boxed{x = \left(\frac{\mu}{1+\mu}\right)}$$

(Block on Block Qs)

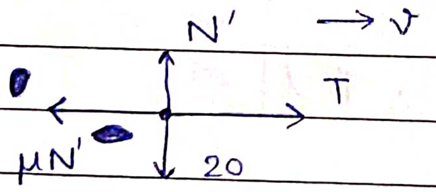
Q)



If blocks move with const. vel., find F.

A)

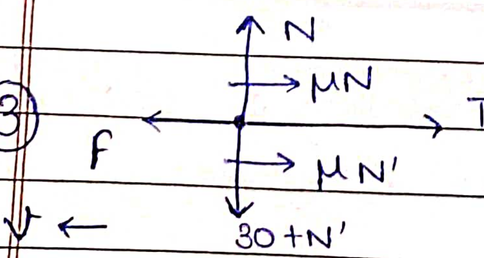
(2)



$N' = 20 \Rightarrow \mu N' = 10$

$\Rightarrow 0 = T - 10 \Rightarrow T = 10$

(3)



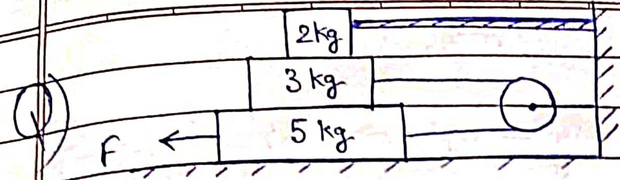
$N = 30 + N' = 50$

$\Rightarrow \mu N = 25$

$$F = T + \mu N + \mu N'$$

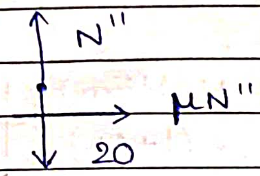
$$\Rightarrow F = 10 + 25 + 10 \Rightarrow \boxed{F = 45}$$

Q)

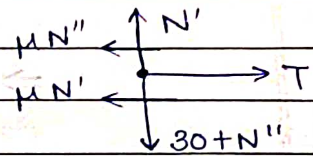


2 kg is fixed
μ at all surfaces 0.5.
Find f if 5kg moves with const. vel.

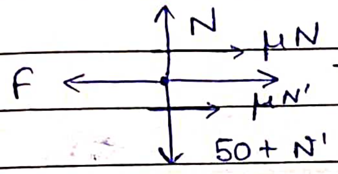
A) (2)



(3)



(5)

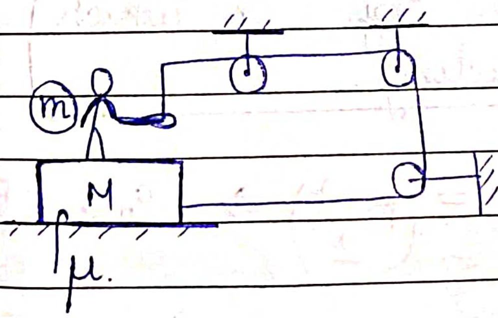


from (2), $N'' = 20$

from (3), $N' = 30 + N''$ $\Rightarrow$ $N' = 50$, $T = \mu N' + \mu N''$ $\Rightarrow$ $T = 35$

from (5), $N = 50 + N'$ $\Rightarrow$ $N = 100$, $f = T + \mu N + \mu N'$ $\Rightarrow$ $\boxed{f = 110}$

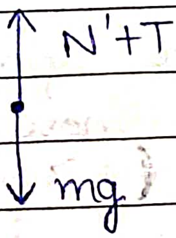
Q)



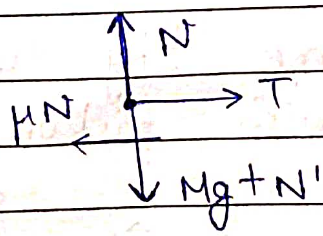
Find force applied by man so that block just moves.

A)

Man



Block



Date:

From Man, $N' + T = mg$

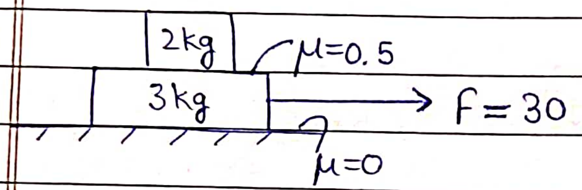
From Block, $N = Mg + N' \Rightarrow N = (M+m)g - T$

$T = \mu N \Rightarrow T = \mu\left[(M+m)g - T\right]$

$$\Rightarrow \boxed{T = \frac{\mu(M+m)g}{(1+\mu)}}$$

(29/6/22)

Q) ☆



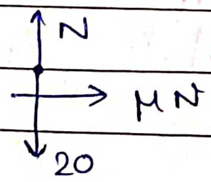
Find acc. of bodies.

A) C-1 : First assume both objs move together

$a = \left(\frac{30}{3+2}\right) \Rightarrow a = 6 \Rightarrow f = 12$

☆ Now check 'a_{max}' of obj. on which ONLY friction is acting.

Check: ②



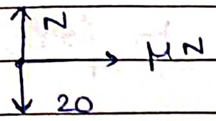
$a_{max.} = \left(\frac{\mu N}{2}\right) \Rightarrow a_{max.} = 5$

$N = 20 \Rightarrow f_{max.} = 10$

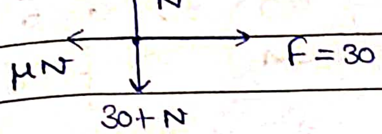
Since 2kg can only move upto $5\,ms^{-2}$ with friction, it can't move with $6\,ms^{-2}$ $\{f > f_{max.}\}$

$\Rightarrow$ Objs. move seperately.

C-2 : Objs. move seperately

② 

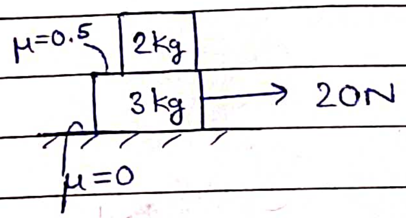
$N = 20 \Rightarrow a_2 = \left(\frac{\mu N}{2}\right) \Rightarrow a_2 = 5$

③ 

$N' = 30 + N \Rightarrow N' = 50$

$a_3 = \left(\frac{f - \mu N}{3}\right) \Rightarrow a_3 = 20/3$

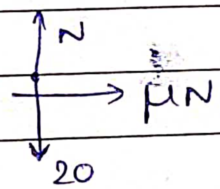
☆ $\boxed{\begin{array}{l} a_{combined} > a_{max.} \Rightarrow \text{Objs. move sep.} \\ f_{combined} > f_{max.} \Rightarrow \text{Objs. move sep.} \end{array}}$

Q) 

find friction acting on 2kg.

A) C-1 : Combined $\quad a = \left(\frac{20}{3+2}\right) \Rightarrow a = 4 \Rightarrow f = 8$

Check :

Now, ② 

$a_{max} = \left(\frac{\mu N}{2}\right) \Rightarrow a = 5$

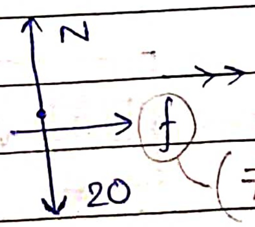
$f_{max.} = 10$.

$N = 20$

Since $a_{combined} < a_{max.} \Rightarrow$ Obj's move together.

$(f < f_{max})$

So,

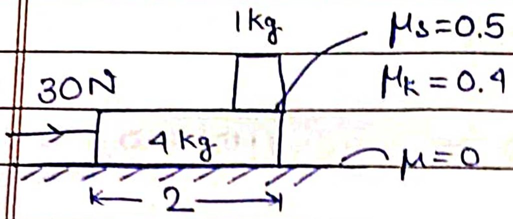
② 

$ma = f$

$\Rightarrow 2 \cdot 4 = f \Rightarrow \boxed{f = 8}$

$(\neq \mu N$ as no rel. motion$)$

Q) Find time taken for block to fall.

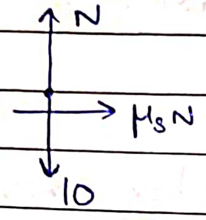


A) C-1: Combined $\quad a = \left(\frac{30}{1+4}\right) \Rightarrow a = 6$

$\Rightarrow f = 6$

Check:

Now, 1



$N = 10 \Rightarrow \mu_s N = 5$

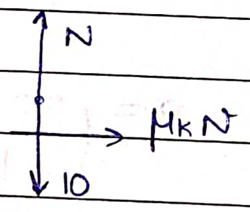
$\Rightarrow a_{max.} = 5 \Rightarrow f_{max.} = 5$

Since, $a_{max.} < a_{combined} \Rightarrow$ Objs. move. sep.

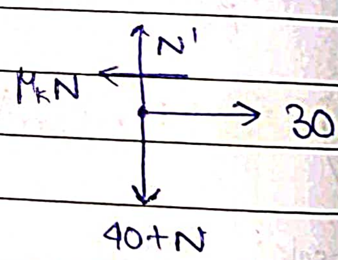
$(f_{max.} < f)$

C-2: Objs. move sep.

1



4



Now, $N = 10 \Rightarrow \mu_k N = 4$

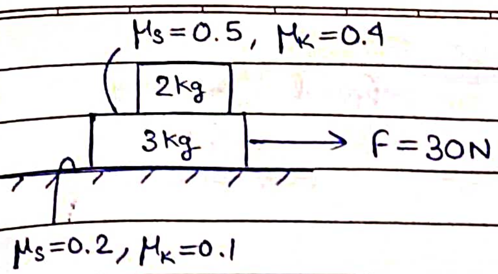
$\Rightarrow a_1 = \left(\frac{4}{1}\right) \Rightarrow a_1 = 4$

$a_{rel.} = 5/2$

$a_4 = \left(\frac{30 - \mu_k N}{4}\right) \Rightarrow a_4 = \left(\frac{13}{2}\right)$

$t = \sqrt{\frac{2l}{a_{rel}}} = \sqrt{\frac{2 \cdot 2}{5/2}} \Rightarrow \boxed{t = \sqrt{1.6}\ s}$

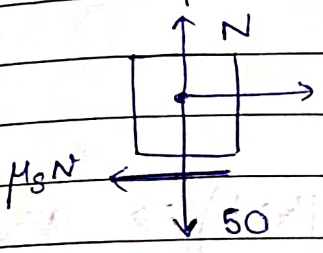
☆ Q)



Find acc. of masses.

A) ☆ Always check 'whether system can even move, using friction from ground'.

Condition for motion.

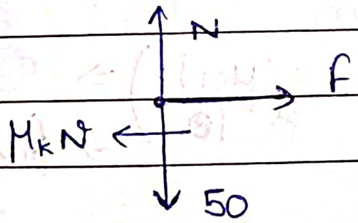


✓ $F > \mu_s N \Rightarrow F > (0.2)(50)$

$\Rightarrow F > 10$ ✓

✓ $N = 50$

C-1 : Objs move together.

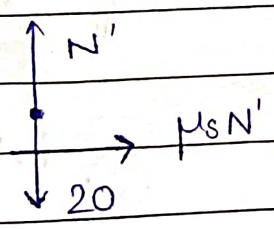


✓ $a = \left(\dfrac{F - \mu_k N}{2+3}\right) \Rightarrow a = 5$

$\Rightarrow f = 10$

✓ $F = 30$

Now, ②



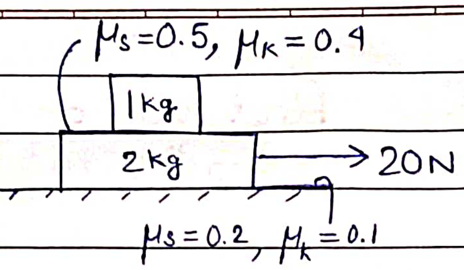
$a_{max.} = \dfrac{(0.5)(20)}{2} \Rightarrow a_{max.} = 5$

$\Rightarrow f_{max.} = 10$

Since, $a_{max.} \geq a_{combined}$ $\Rightarrow$ Objs. move together with
$(f_{max} \geq f)$ $\boxed{a = 5}$

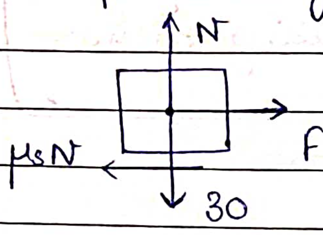
Date: ______ Page

Q)



find acc. of masses.

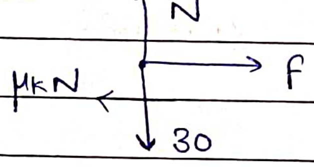
A) Condt for moving,



$f > \mu_s N \Rightarrow f > (0.2)(30)$

$\Rightarrow f > 6$ ✓

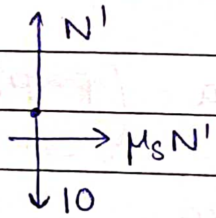
C-1 : Combined



$a = \left(\dfrac{f - \mu_k N}{1+2}\right) \Rightarrow a = \left(\dfrac{17}{3}\right)$

$\Rightarrow f = 17/3$

Now, ①



$a_{max.} = \left(\dfrac{\mu_s N'}{1}\right) \Rightarrow a_{max.} = 5$

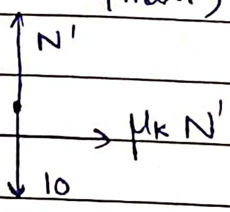
$\Rightarrow f_{max} = 5$

Since $a_{combined} > a_{max.} \Rightarrow$ Objs move sep.

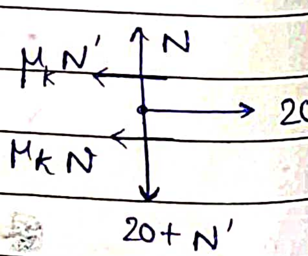
$(f > f_{max.})$

C-2 :

①



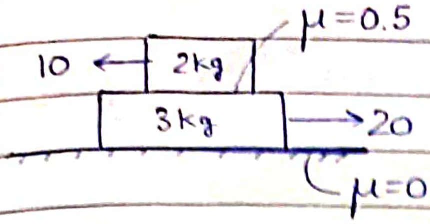
②



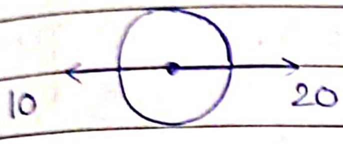
$a_1 = \left(\dfrac{\mu_k N'}{1}\right) \Rightarrow a_1 = 4$

$a_2 = (20 - \mu_k N' - \mu_k N) / 2$

$\Rightarrow \boxed{a_2 = \left(\dfrac{13}{2}\right)}$

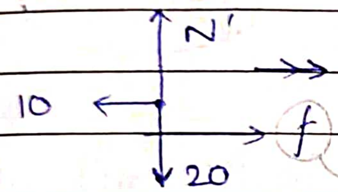
☆ Q)  find acc. of blocks.

A) Let's assume blocks move together.



$a = \left(\frac{20-10}{2+3}\right) \Rightarrow a = 2$

In this case we find friction on 2kg

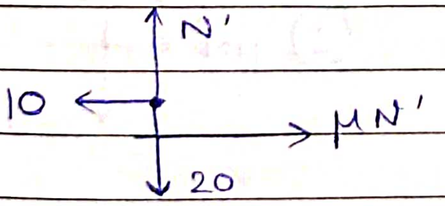
(2) 

$f - 10 = 2 \cdot 2 \Rightarrow f = 14$

($\neq \mu N'$ as no rel. motion)

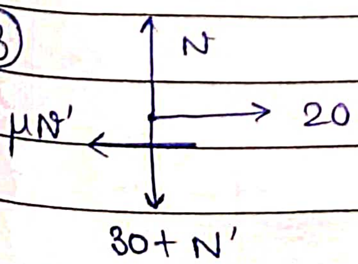
But max. friction on 2kg can be $\mu N = 10$.

$\Rightarrow$ Blocks move sep.

(2) 

$a_2 = \left(\frac{\mu N' - 10}{2}\right)$

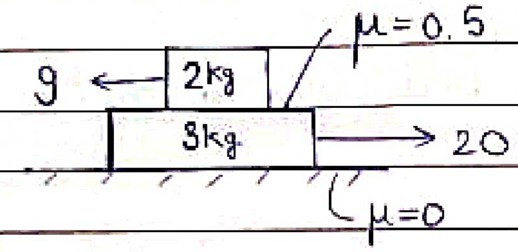
$\Rightarrow a_2 = 0$

(3) 

$a_3 = \left(\frac{20 - \mu N'}{3}\right)$

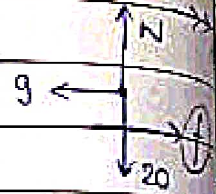
$\Rightarrow a_3 = 10/3$

Q) Find acc. of blocks



A) Assume both move together.

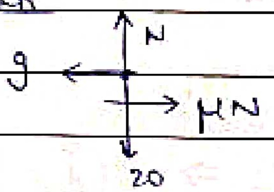
So, $a = \left(\frac{20-9}{2+3}\right) \Rightarrow \boxed{a = 2.2}$



$\Rightarrow \boxed{f = 13.4}$

($\neq \mu N$ as no rel. motion)

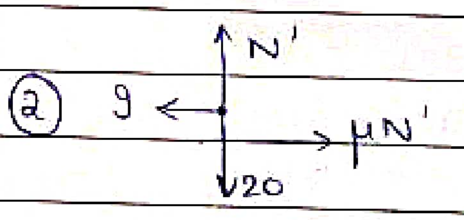
Check:

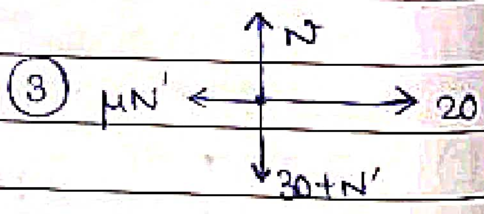


$f_{max.} = \mu N \Rightarrow \boxed{f_{max.} = 10}$

$f > f_{max.} \Rightarrow$ Blocks can't move together

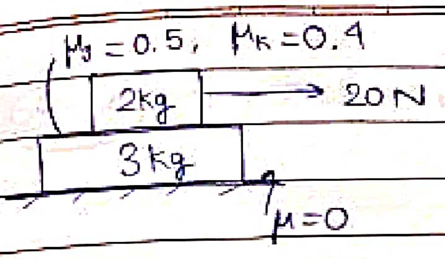
If they move sep. then,





$\Rightarrow \boxed{a_2 = 1/2}$ $\Rightarrow \boxed{a_3 = 10/3}$

Q)



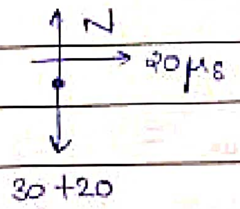
find acc. of blocks.

A) C-1: Combined

$a = \left(\frac{20}{3+2}\right) \Rightarrow a = 4$

$\Rightarrow f = 12$

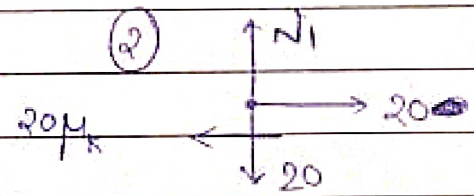
Check:

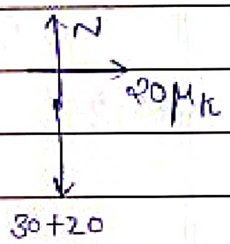


$f_{max} = 20\mu_s \Rightarrow f_{max.} = 10$

Since $f > f_{max.} \Rightarrow$ Blocks move sep.

C-2: Sep.

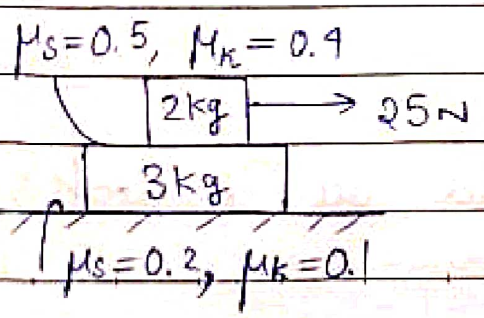




$a_2 = \left(\frac{20 - 20\mu_K}{2}\right)$ $\quad a_3 = \left(\frac{20\mu_K}{3}\right)$

$\Rightarrow \boxed{a_2 = 6}$ $\quad \boxed{a_3 = 2.67}$

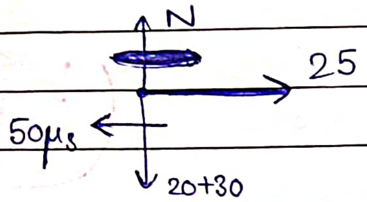
Q)



find acc. of blocks

A) C-1 : Combined $a = \left(\frac{25}{5}\right) \Rightarrow a = 5$

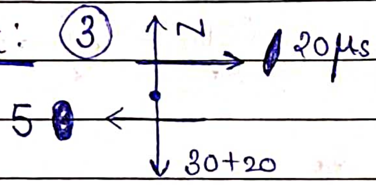
Check : Motion ?



$25 > 10 = 50\mu_s$

$\Rightarrow$ Motion ✓

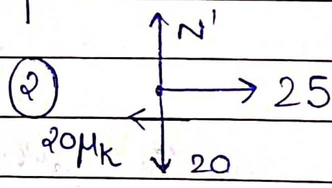
Check : 3

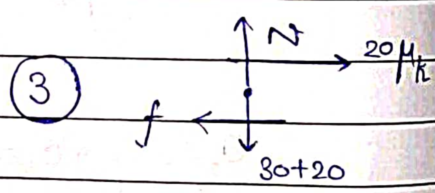


$a_{max.} = \left(\frac{10-5}{3}\right) \Rightarrow a_{max} = 5$

Since $a > a_{max.} \Rightarrow$ Obj move sep.

C-2 : Sep.





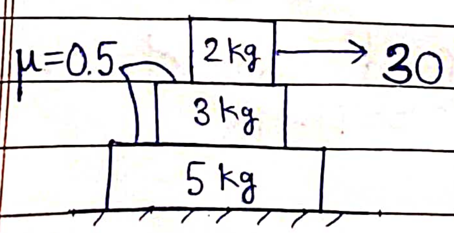
$a_2 = \left(\frac{25-8}{2}\right)$

$f = 8 < f_{max.} = 10$

$\Rightarrow \boxed{a_2 = 8.5}$

$\Rightarrow \boxed{a_3 = 0}$

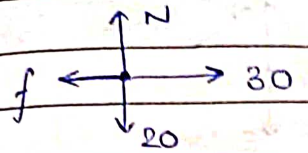
Q)



Find acc. of blocks.

A) C-1: Combined $\quad a = \left(\dfrac{30}{2+3+5}\right) \Rightarrow a = 3$

(2)



$f = 30 - 2\cdot 3 \Rightarrow f = 24$

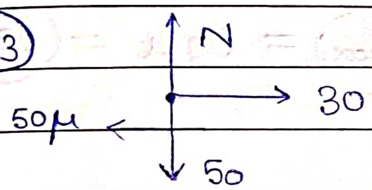
Check: $f_{max.} = 20\mu \Rightarrow f_{max} = 10$

$f_{max} < f \Rightarrow$ Obj NOT combined.

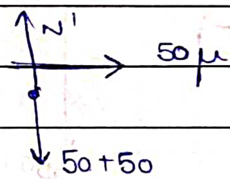
3

C-2: (2+3) and 5 sep.

(2+3)



(5)



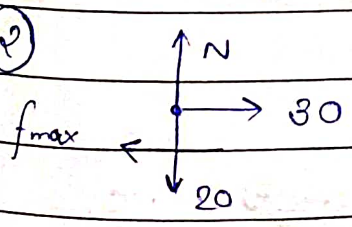
$$a_2 = a_3 = \left(\frac{30 - 50\mu}{5}\right) = 1$$

$\cancel{f_{2,3} = \dfrac{30 - 2\cdot 1}{2}}$ $\quad 2a_2 = (30 - f_{2,3})$

$\Rightarrow f_{2,3} = 28$

Check: 2 & 3 together?

(2)



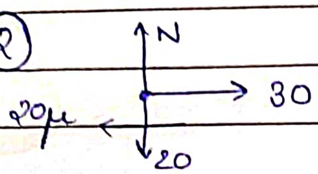
$f_{max.} = 20\mu \Rightarrow f_{max} = 10$

Since $f_{max} < f_{2,3} \Rightarrow$ 2 & 3 sep.

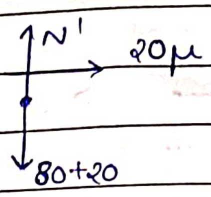
Date:

C-3 : 2 & (5+3) sep.

(2)



(3+5)

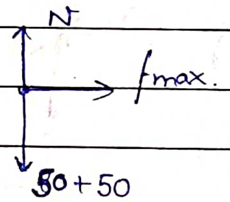


$$a_3 = a_5 = \left(\frac{20\mu}{8}\right) = \left(\frac{10}{8}\right)$$

$$f_{5,3} = 5\left(\frac{10}{8}\right) \Rightarrow f_{5,3} = 25/4$$

Check: 3 & 5 combined?

(5)



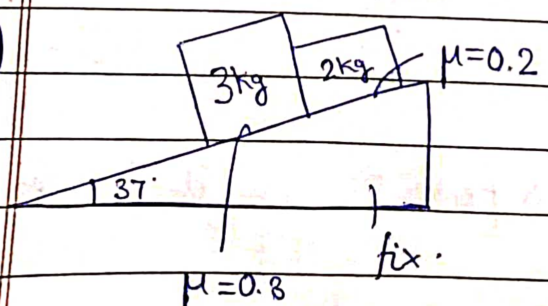
$f_{max} = 50\mu = 25$

Since $f_{5,3} < f_{max.} \Rightarrow$ Combined

So final soln,

$\boxed{a_2 = 10}$, $\boxed{a_3 = a_5 = 1.25}$

Q)



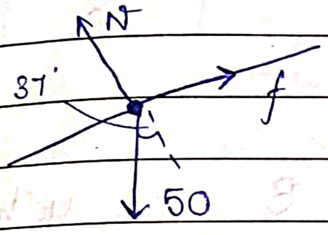
Find ~~net~~ acc. of blocks and Normal force b/w them.

A) C-1: Assume blocks move sep.

$a_3 = g(s_{37°} - (0.3)\,c_{37°})$; $a_2 = g(s_{37°} - (0.2)\,c_{37°})$

Obviously $a_3 < a_2 \Rightarrow$ Physically Impossible.

C-2: Combined



$$f = 30\left(\frac{4}{5}\right)(0.3) + 20\left(\frac{4}{5}\right)(0.2)$$

$$\Rightarrow \boxed{f = 10.4}$$

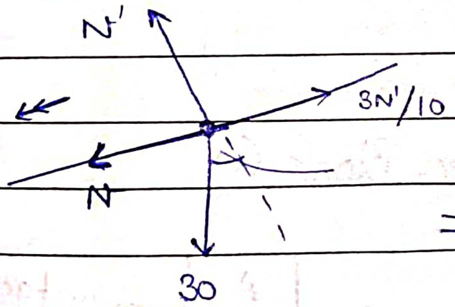
Combined

$$f_{\text{Supporting Motion}} = 50\,s_{37°} = 30$$

$$\Rightarrow a = \left(\frac{30 - 10.4}{5}\right) \Rightarrow \boxed{a = 3.92\ ms^{-2}}$$

3

a



$$3(3.92) = N + 30\,s_{37°} - \left(\frac{3}{10}\right)(30)\,c_{37°}$$

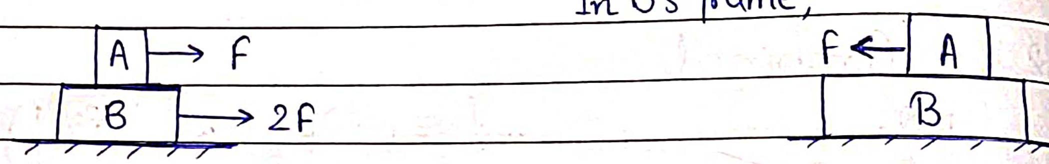
$$\Rightarrow 11.76 = N + 18 - 7.2$$

$$\Rightarrow \boxed{N = 0.96}$$

☆ Finding Dirx^n of friction

Let ∃ 2 objs. A & B. friction by B on A ki dirx^n ke liye, B ke frame me dekho A kahan ja raha hai.

In B's frame,

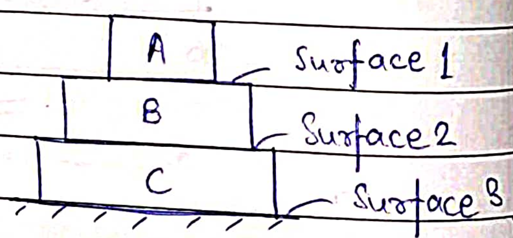


A left jana chahata hai. B usse udhar jane nahi deta.

⇒ friction by B on A towards RIGHT

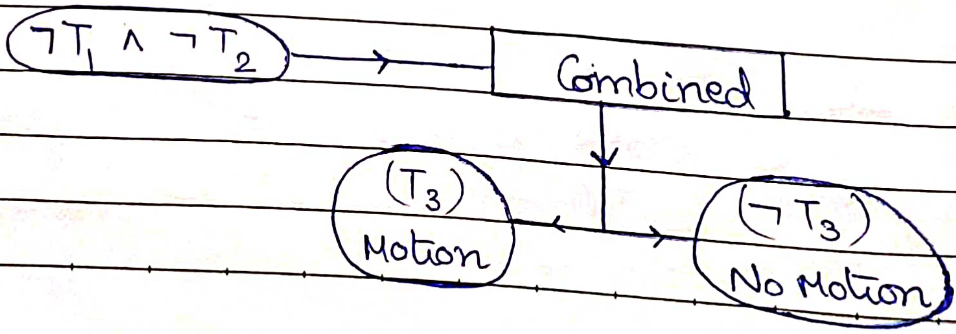
☆ Cases in Block on Block Q

First assume all objs move togther.



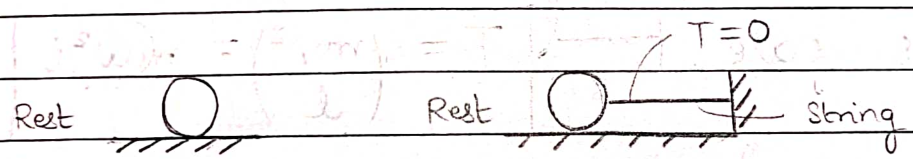
T_1 = Rel. Motion at Surface 1 ?
T_2 = Rel. Motion at Surface 2 ?
T_3 = Rel. Motion at Surface 3 ?

Check these conditions

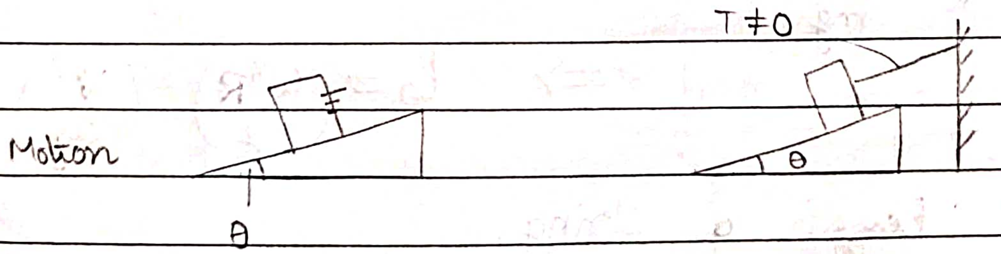


- T_1 → (Reduced to 2 body problem) → $\boxed{A \text{ \& } (B+C)}$
- T_2 → (Reduced to 2 body problem) → $\boxed{(A+B) \text{ \& } C}$
- $T_1 \wedge T_2$ → $\boxed{\text{ALL sep.}}$

☆ If an obj. is naturally at rest, then if a new string is connected to it and fixed wall, no tension in string.



☆ If obj. is naturally moving, then if a new string is connected to it and fixed wall, tension $\neq 0$



$\theta > \tan^{-1}(\mu)$

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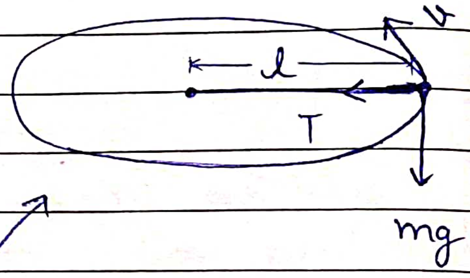
Date:

Circular Motion —

Horizontal Circular Motion —

Centripetal Force (F_c):
Net force towards centre of circle.

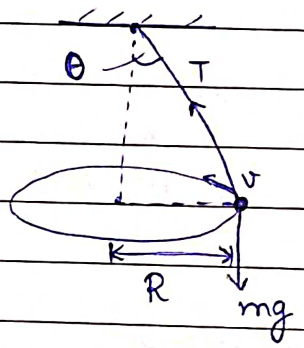


$$\boxed{F_c = \left(\frac{mv^2}{R}\right)}$$

In this case,

$$\boxed{T = \left(\frac{mv^2}{l}\right) = m\omega^2 l}$$

Conical Pendulum



Here, $T\cos\theta = mg$

& $T\sin\theta = \left(\frac{mv^2}{R}\right)$

$$\Rightarrow \tan\theta = \left(\frac{\omega^2 R}{g}\right) = \left(\frac{v^2}{Rg}\right)$$

If l length of string,

$$l\sin\theta = R \Rightarrow \tan\theta = \left(\frac{\omega^2 l\sin\theta}{g}\right)$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{g}{l\cos\theta}}}, \quad \boxed{T = 2\pi\sqrt{\frac{l\cos\theta}{g}}}$$

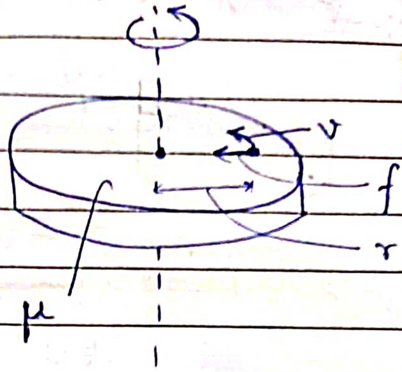
Only friction can give centipetal acc.

$$f = m\omega^2 r$$

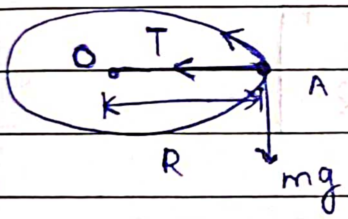
$$f \leq \mu mg$$

$$\Rightarrow \boxed{r \leq \left(\frac{\mu g}{\omega^2}\right)}$$

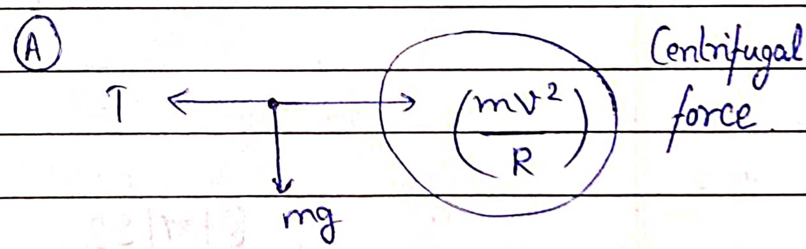
Rotating Table/Disc

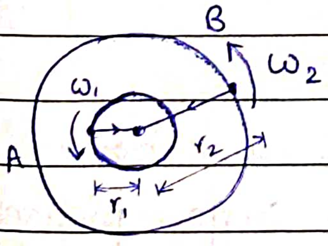


Centrifugal force : Pseudo force on other obj. when observer in rotating body's frame.

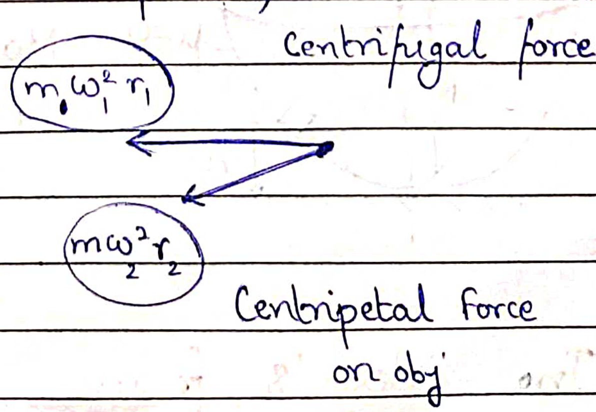


In A's frame,



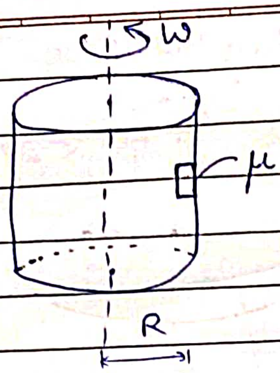


In A's frame,



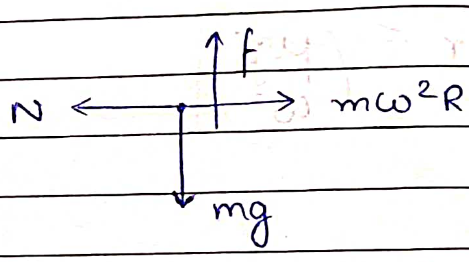
Date:

Q)



Find ω s.t. block doesn't move wrt. wall

A) In obj's frame,



$f = mg$

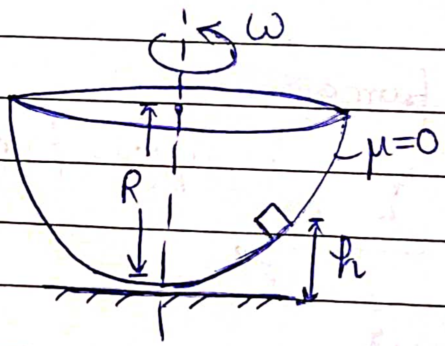
$N = m\omega^2 R$

$f \le \mu N$

$\Rightarrow \boxed{\omega \geq \sqrt{\dfrac{g}{\mu R}}}$

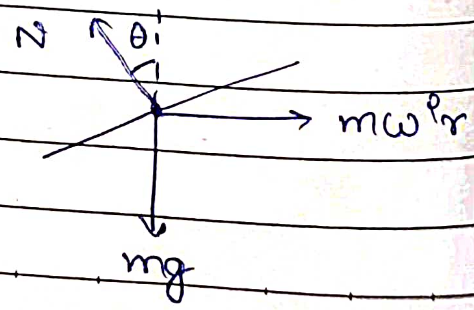
8/7/22

Q)



Mass at rest wrt bowl. Find ω, if surface frictionless

A) In block's frame,

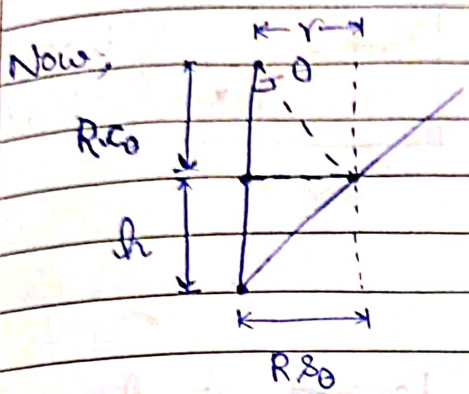


$N c_\theta = mg$

$N s_\theta = m\omega^2 r$

$$\Rightarrow \omega^2 = \left(\frac{g t_\theta}{r}\right)$$ r - Radius of revolution

Now,



$$R c_\theta + h = R \Rightarrow c_\theta = \left(\frac{R-h}{R}\right)$$

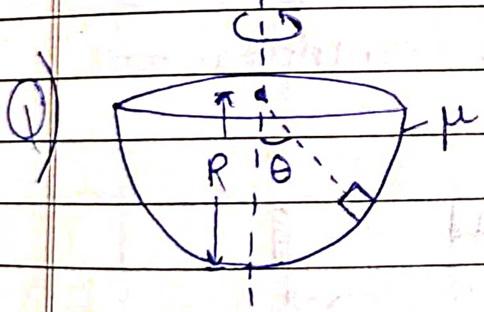
$$r = R s_\theta$$

$$\Rightarrow \omega^2 = \left(\frac{g t_\theta}{R s_\theta}\right) = \left(\frac{g}{R c_\theta}\right) = \left(\frac{g}{R-h}\right)$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{g}{R-h}}}$$

---

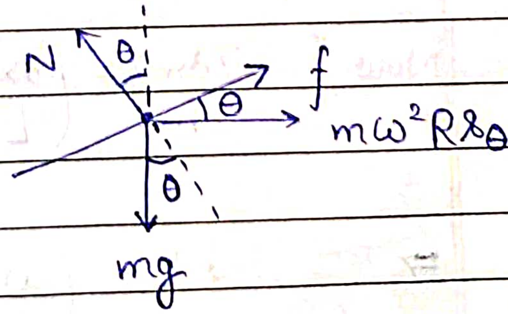
1)



Mass at rest wrt. bowl. Find, ω, if surface fric. coeff. μ min. & max.

A) In block's frame,

$$f = |mg s_\theta - m\omega^2 R s_\theta c_\theta|$$



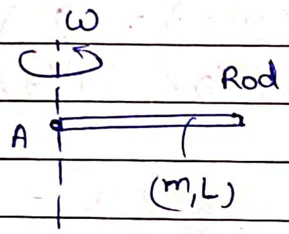
$$N = mg c_\theta + m\omega^2 R s_\theta^2$$

Now, $f \leq \mu N \Rightarrow |g s_\theta - \omega^2 R s_\theta c_\theta| \leq \mu (g c_\theta + \omega^2 R s_\theta^2)$

$$\Rightarrow \left(\frac{g}{R s_\theta}\right)\left(\frac{s_\theta - \mu c_\theta}{c_\theta + \mu s_\theta}\right) \le \omega^2 \le \left(\frac{g}{R s_\theta}\right)\left(\frac{s_\theta + \mu c_\theta}{c_\theta - \mu s_\theta}\right)$$

$$\Rightarrow \boxed{\omega_{min} = \sqrt{\left(\frac{g}{R s_\theta}\right)\left(\frac{t_\theta - \mu}{1+\mu t_\theta}\right)}}, \quad \boxed{\omega_{max.} = \sqrt{\left(\frac{g}{R s_\theta}\right)\left(\frac{t_\theta + \mu}{1-\mu t_\theta}\right)}}$$

☆ Q)

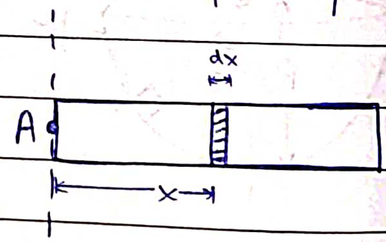


Find tension at A.

A) We integrate, wrt dist. from A.

At each pt. T acts as centripetal force.

Since T decrease from A to end,

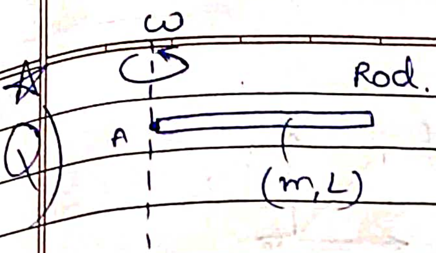


$$-dT = dm\,\omega^2 x$$

Now, $\left(\frac{dm}{m}\right) = \left(\frac{dx}{L}\right) \Rightarrow dm = \left(\frac{m\,dx}{L}\right)$

$$\Rightarrow -dT = \left(\frac{\omega^2 x m}{L}\right) dx \Rightarrow -\int_T^0 dT = \int_0^L \left(\frac{\omega^2 m}{L}\right) x\,dx$$

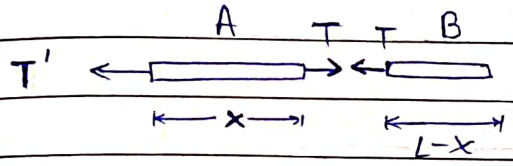
$$\Rightarrow \boxed{T = \left(\frac{\omega^2 m L}{2}\right)}$$



Q) Find tension at a dist. 'x' from A.

A) Like in prev. Q, $-dT = \left(\frac{\omega^2 m}{L}\right) x$

We need to choose limits carefully.



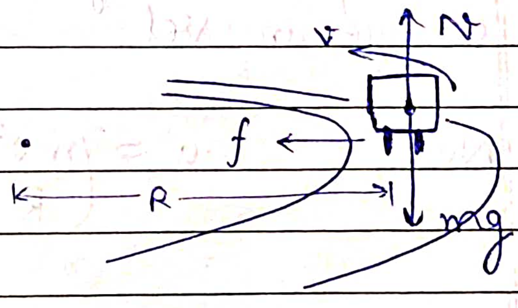
What we require is T

⇒ Centripetal force of B!

$$\Rightarrow \int_T^0 -dT = \left(\frac{\omega^2 m}{L}\right)\int_x^L x\,dx \Rightarrow \boxed{T = \left(\frac{m\omega^2}{2L}\right)(L^2 - x^2)}$$

Banking of Roads —

Flat Road (Uniform vel.)



$$f = \left(\frac{mv^2}{R}\right)$$

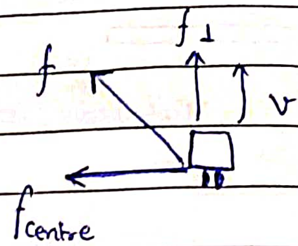
Now for turning, $f \leq \mu N$, $N = mg$

$$\Rightarrow \boxed{v \leq \sqrt{\mu R g}}$$

If $v > \sqrt{\mu R g}$ ⇒ Car will skid.

Date: ______ Page

Flat Road (Accelerating)



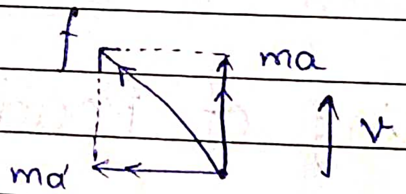
Friction $\leq \mu mg$

A comp. towards centre = Centripetal force

A comp. along vel. = Acc. force

Q) A body moving on flat curved road of radius R. Its ~~vel.~~ speed is ~~inc~~ changing at rate of 'a'. Find max. vel. of body w/o skiding.

A) Friction is the only force causing change in vel.



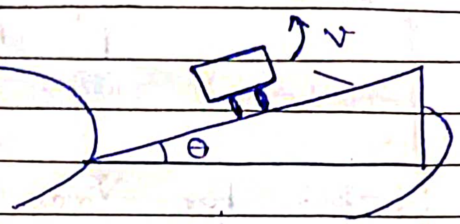
Now, $ma' = \left(\frac{mv^2}{R}\right)$; $f \leq \mu N = \mu mg$

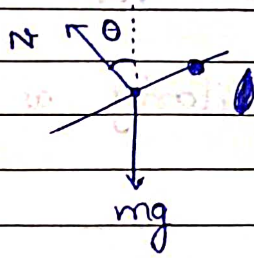
$\Rightarrow \mu mg \geq f = \sqrt{(ma)^2 + (ma')^2}$

$\Rightarrow (\mu g)^2 \geq a^2 + \left(\frac{v^4}{R^2}\right) \Rightarrow \boxed{v \leq \left(R^2\left((\mu g)^2 - a^2\right)\right)}$

Banked Road (w/o friction)

$N s_\theta$ acts as centripetal force.





$$\Rightarrow \quad N s_\theta = \left(\frac{mv^2}{R}\right)$$

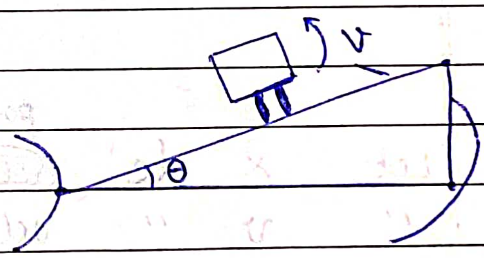
$$N c_\theta = mg$$

$$\Rightarrow \quad t_\theta = \left(\frac{v^2}{Rg}\right) \Rightarrow \boxed{v = \sqrt{Rg t_\theta}}$$

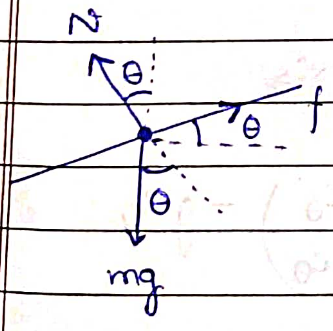
→ At this vel., $f = 0$

Banked Road (with friction)

Both v_{min} & v_{max} exist.



C-1 : for v_{min}, friction acts up



$$N c_\theta + f s_\theta = mg$$

$$N s_\theta - f c_\theta = \left(\frac{mv^2}{R}\right)$$

$$\Rightarrow \quad f = mg s_\theta - \left(\frac{mv^2}{R}\right) c_\theta \quad , \quad N = mg c_\theta + \left(\frac{mv^2}{R}\right) s_\theta$$

Obviously $f \leq \mu N \Rightarrow mg(s_\theta - \mu c_\theta) \leq \left(\frac{mv^2}{R}\right)(c_\theta + \mu s_\theta)$

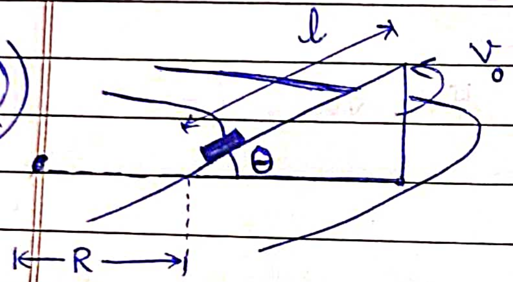
Date:

$$\Rightarrow \quad v \geq \sqrt{gR\left(\frac{t_\theta - \mu}{1+\mu t_\theta}\right)} \Rightarrow \boxed{v_{min} = \sqrt{gR\left(\frac{t_\theta - \mu}{1+\mu t_\theta}\right)}}$$

C-2 : For $v_{max.}$, friction acts down.

Similarly as C-1, $\boxed{v_{max.} = \sqrt{gR\left(\frac{t_\theta + \mu}{1-\mu t_\theta}\right)}}$

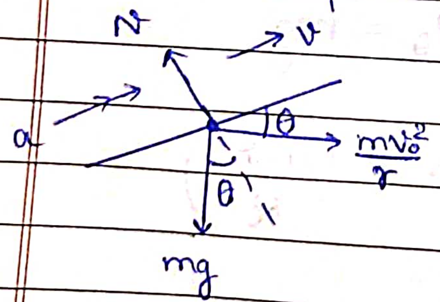
Q)



Incline plane moving. If $\mu = 0$, find time taken by mass to reach top.

A) Let x be ~~vel~~ post. of obj along incline.
" v " vel. " " " "

In block's frame,



$$r = R + x c_\theta$$

$$\Rightarrow a = \left(\frac{v_0^2 c_\theta}{R + x c_\theta}\right) - g s_\theta$$

$$\Rightarrow v\,dv = \boxed{\left(\frac{v_0^2 c_\theta}{R + x c_\theta}\right) - g s_\theta}\,dx$$

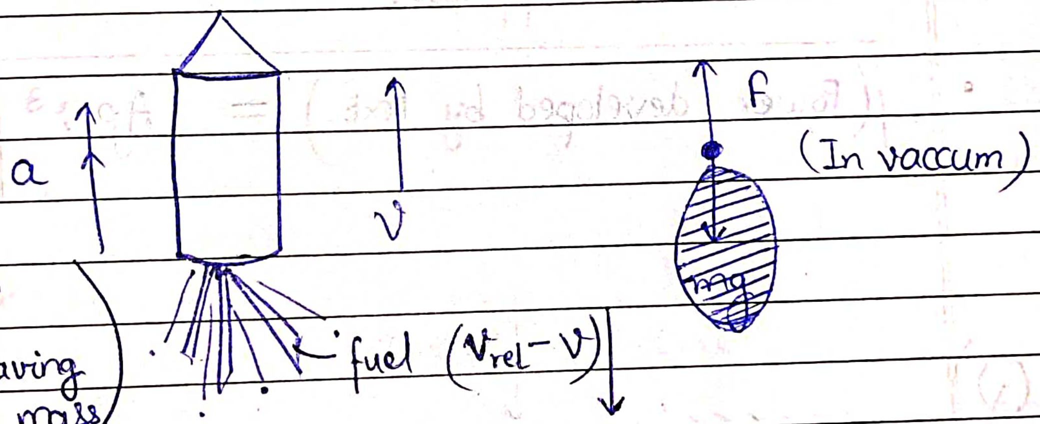
$$\Rightarrow \left(\frac{v^2}{2}\right) = \left(v_0^2\right) \ln\left|\frac{R + x c_\theta}{R}\right| - \left(g s_\theta\right) x$$

$$\Rightarrow \quad v = \sqrt{(2v_0^2)\ln\left|\frac{R + x c_0}{R}\right| - (2g s_0)x}$$

write $v = \left(\frac{dx}{dt}\right)$. Solve for x in terms of t.

Variable Mass —

Rocket.
(in Vaccum)



(v_{rel} - Vel. of mass leaving w.r.t. main mass)

Since momentum of rocket same,

$$p = mv = (m - \Delta m)(v + \Delta v) - (\Delta m)(v_{rel} - v) \Rightarrow mv = mv - v\Delta m + m\Delta v - \Delta m \Delta v + v\Delta m - v_{rel}\Delta m$$

before — after

$\Delta m \Delta v$: neglect

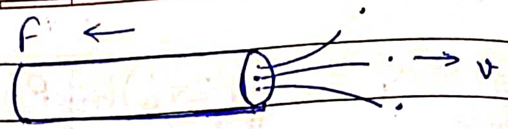
$$\Rightarrow m\Delta v = (v\Delta m - v\Delta m) + v_{rel}\Delta m$$

$$\Rightarrow v_{rel}\left(\frac{\Delta m}{\Delta t}\right) = m\left(\frac{\Delta v}{\Delta t}\right) \longrightarrow m\left(\frac{dv}{dt}\right) = v_{rel}\left(\frac{dm}{dt}\right)$$

$$\Rightarrow \boxed{F = v_{rel}\left(\frac{dm}{dt}\right)}$$

$(dm/dt) < 0 \Rightarrow F$ opp. v_{rel}

$(dm/dt) > 0 \Rightarrow F \parallel v_{rel}$



Pipe

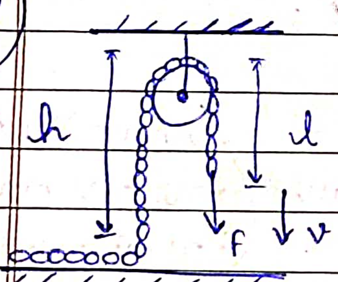
If fluid flow at const. rate/ Time considered Very Small.

$dm = \rho\, dV \Rightarrow dm = \rho A\, dx$

$\Rightarrow \left(\frac{dm}{dt}\right) = \rho A \left(\frac{dx}{dt}\right) \Rightarrow v\left(\frac{dm}{dt}\right) = f = \rho A v^2$

- To stop pipe from coming back, $f = \rho A v^2$ must be applied.
- $\boxed{\left(\text{Power developed by } f_{ext.}\right) = A\rho v^3}$

☆ Q)



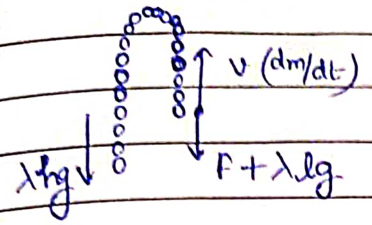
Find force f needed to move chain down with const. vel. Mass per unit length = λ.

A) Chain of length h has const. mass as h const.
" " " l " variable " " l variable

Upward force on l $= v\left(\frac{dm}{dt}\right) = v\left(\frac{dm}{dl}\right)\left(\frac{dl}{dt}\right)$

$= v^2\lambda$

Consider whole chain. Since equilibrium,



$$\Rightarrow \quad f + \lambda l g - v(dm/dt) = \lambda h g$$

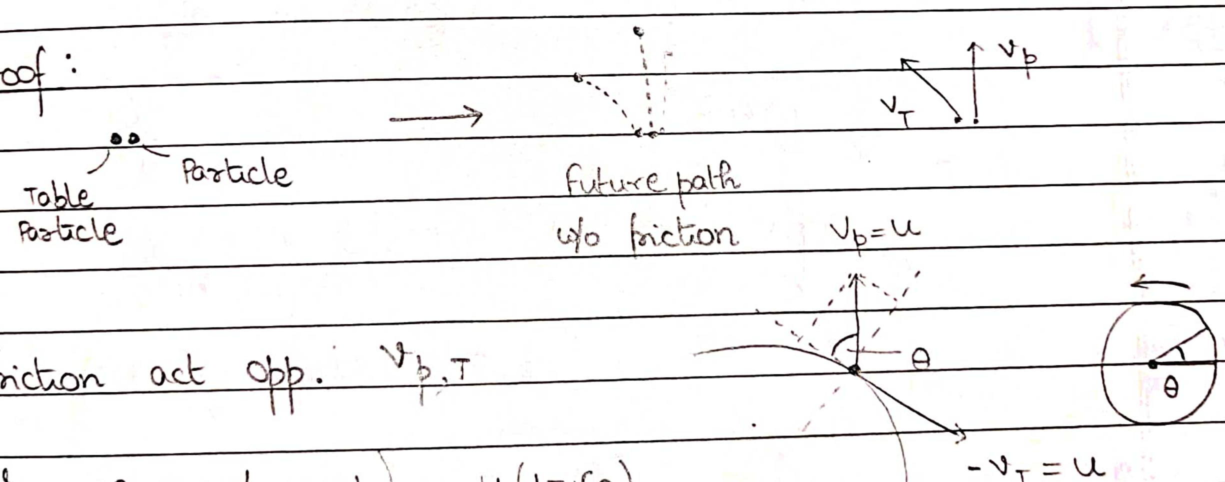
$$\Rightarrow \quad f = \left(\lambda h + \frac{\lambda v^2}{g} - l\lambda\right) g$$

$$\Rightarrow \quad \boxed{f = (\lambda)(hg + v^2 - lg)}$$

Imp. –

1) Why friction in rotating table pt. inwords?

Proof :



Friction act opp. $v_{p,T}$

$\left(v_{p,T} \text{ Comp. tangent to table} \right) = u(1 - c_\theta)$

$\left(v_{p,T} \text{ comp normal to table} \right) = u s_\theta$

Now, $(1 - c_\theta) \to 0$ much faster than $(s_\theta \to 0)$ when $(\theta \to 0)$ [By Derivative]

$\Rightarrow$ Friction acts inwards!

Date: ______ Page:

2) In variable mass, $v\left(\frac{dm}{dt}\right)$ along vel. if mass dec. ; and opp. to vel. if mass inc.